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Can a path-dependent strategy outperform a path-independent strategy?

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ABSTRACT

The question of whether a path-independent strategy can outperform a path-dependent one has given rise to an interesting debate within the finance literature. This paper uses a protective put as an example and shows that by embedding a cost-down method into our approach, a path-dependent strategy may well outperform a path-independent one. The simulation results illustrate that our proposed protective put outperforms a classical protective put, exhibiting superior capabilities in terms of capturing upside potential, leading to higher Sharpe ratios and Sortino ratios, and avoiding insolvency. The results obtained from our sensitivity analysis provide further confirmation of the robustness of our simulation findings.

1. Introduction

While the payoffs of some derivatives are determined only by the terminal values of their underlying assets (e.g., European options), for others (e.g., American and Asian options), the payoffs are determined by the intermediate values of the underlying assets. The payoffs of the former are called path-independent, whereas those of the latter are known as path-dependent. In order to attract investors, a financial institution may design complicated payoff structures and utilize different investment strategies to achieve their goals, such as a protective put. The question of whether a path-independent strategy can outperform a path-dependent one has given rise to an interesting debate within the finance literature.

Cox and Leland (2000) argued that if investors were concerned only with their final wealth, then an efficient strategy would not be constrained by intermediate values. As noted by Leland (1985), transaction costs tend to invalidate the Black–Scholes option pricing arbitrage argument, since such transaction costs can, in practice, make continuous trading ruinously expensive, regardless of how small the transaction costs may be as a percentage of the overall turnover level. An option-based dynamic strategy would thus tend to be infinitely costly. Indeed, this general conclusion has since become a golden rule among certain investors, such as option-based portfolio insurance devotees, and has thus been in applied in their path-independent strategies.

Since a path-independent product only takes into account the terminal value of underlying assets, this means that a path-independent strategy is not available for American options, and thus alternative designs are needed for such products. For example, the inclusion of the intermediate value of the wealth that occurs during the life of a contract has also attracted considerable academic interest. Barraquand and Pudet (1996) included path-dependent variables in the Black–Scholes valuation framework, which reduces degenerate diffusion equations to lower-dimensional non-degenerate diffusion equations. They then applied the results to path-dependent options and proposed a

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'forward shooting grid' (FSG) method which is capable of dealing with the early exercise condition of American options. Karoui, Jeanblanc, and Lacoste (2005) subsequently addressed the value of American guarantees based on American puts, and also developed a path dependent self-financing strategy, extending the optimum results to a general utility function within strategy.

In practice, however, in contrast to traditional life insurance products, the fundamental principle of equity-linked contracts is that the risk for the insurer is non-diversifiable across individual policyholders. That is, any market collapse – such as those which occurred in 1987 and 1998 – would render the insurance company simultaneously liable under the guarantees of all its expiring policies (Brennan & Schwartz, 1976). Options are thus applied in the design of guaranteed contracts in order to avoid insolvency.

Guaranteed equity-linked contracts are generally seen as being as competitive as high interest rate products or bank products within financial markets (Hipp, 1996). A common feature among such products is that investors are becoming increasingly concerned about the payoff based upon the value of the stock market during the overall life of the contract, and thus protective-put-based guaranteed contracts can attract more investors to the market.

In the simplest situation, the payoff of a guaranteed contract at expiration is equal to that of a European call option plus the guaranteed amount, with the pricing problem in this kind of scheme being solved by using the Black–Scholes theory (Brennan & Schwartz, 1976; Boyle & Schwartz, 1977). Using put-call parity, those guarantees that are based upon call options can be rewritten as a combination of stocks and put options (i.e., a protective put), which provides the theoretical basis for the design of a guaranteed equity-linked contract in this paper.

A protective put is comprised of one stock and one European put, and, by definition, a European put only counts for the probability distribution at the expiration of the contracts, and so the classical protective put is a path-independent strategy. We therefore use the classical protective put as a representative path-independent strategy in our examination as to whether such an approach can outperform a path-dependent one.

A protective put leads to a portfolio return with a high peak and truncated normal distribution, due to the fact that the strike price of a protective put squeezes the payoffs below this. Within the statistical literature, it was demonstrated by Law and Kelton (1991) and Kececioglu (1991) that the mean (variance) of a truncated normal distribution is an increasing (decreasing) function of the truncation point. Barr and Sherrill (1999) further showed that this is also true for non-standard normal populations. The truncation point is thus related to the Sharpe ratio in a non-linear option-like return distribution, with the strike price being the truncation point of this type of return distribution for option-based strategies.

A clear example of this was provided by Lhabitant (2000), who explored the relationships between strike price levels and Sharpe ratios in covered call and protective put strategies, and found that the Sharpe ratios were negatively related to strike price levels in a protective put, whereas they were positively related to strike price levels in a covered call.

The Sharpe ratio is a widely used criterion for evaluating portfolio performance, especially among risk-averse investors. A strategy that leads to a higher Sharpe ratio will be considered as better than the alternatives, because of its capability to obtain greater returns with respect to the risk taken. In addition to risk, investors are also concerned with losses, and the Sortino ratio can be used to evaluate how well a particular strategy is able to avoid these. In this context, a strategy with a higher Sortino ratio can be seen as a better one, because it is better able to avoid large losses.

As stated above, a protective put leads to a truncated normal distribution with a high peak at the truncation point, which corresponds to the strike price of the put option. The statistical literature suggests that, for a protective put, moving the strike price upward may increase (reduce) the mean (variance), and as such, an upwardly moving strike price may lead to higher Sharpe ratios. Furthermore, from a loss-aversion perspective, moving the strike price upward will also lead to higher Sortino ratios.

Both risk- and loss-averse performance measures imply that moving strike prices upward in a protective put during the life of a contract will lead to higher performance measures. However, in equilibrium, when moving the strike price to a higher level in a protective put, the additional benefit obtained is exactly offset by its cost. That is to say, higher performance measures cannot be obtained by manipulating strike prices in a static protective put.

The assumption that investors are concerned only with their final wealth is quite extreme, and perhaps, somewhat unrealistic. With regard to products such as rolling guarantee funds, investors place money into the contract for a required period of time (for example, three years). However, certain circumstances may require a withdrawal before maturity, and in such situations it is clear that intermediate guarantees are of some value to investors.

The classical protective put asserts that within an equilibrium market there are no opportunities to obtain excess returns by manipulating variables, such as the choice of strike prices. As a result, investors will invariably apply a static approach, e.g., a protective put, through the use of an at-the-money put, and then hold the portfolio until its expiration. This is the most common way for a protective put to be utilized in the market. The opportunity of capturing the intermediate value that occurs during the life of a contract is then ignored. Therefore, both assumptions of investors being only concerned with their final wealth and being unable to obtain excess returns are strong. However, the possibility of getting higher Sharpe and Sortino ratios by manipulating strike prices motivates us to propose a path-dependent approach as a competing strategy with the classic protective put.

If we can obtain an additional expected return at a lower cost, then we can get a higher performance measure. Fortunately, since a put premium is negatively related to the stock price, the inclusion of down-and-in barrier 'knock-in' puts within a static protective put (i.e., the formation of a dynamic protective put) makes it possible to obtain the same expected return of alternative strike prices at lower costs.

Although the finance literature contains a number of theoretical discussions as to which strategy, path-independent or path-dependent, is superior, to the best of our knowledge, no empirical evidence has been presented to help resolve this issue. In this paper we propose that a path-dependent strategy, using a dynamic protective put as an example, could outperform a path-independent strategy.

We follow the Merton (1976) model by applying 2000 stock price simulations to compare the evolution of return distributions, statistics and performance measures between the alternative strategies. The simulation results indicate that, as compared to the return distribution obtained with a static protective put, a dynamic strategy leads to a gradual upward movement in returns. Specifically, a dynamic protective put results in greater possibilities of higher returns and lower losses than a static protective put. A dynamic protective put therefore outperforms a static protective put in terms of higher Sharpe and Sortino ratios. The results obtained from our sensitivity analysis confirm the robustness of the findings obtained from our simulations.

Our paper makes important contributions to the existing literature in three respects. Firstly, in addition to using continuous models, we also derive discrete methods of estimating the expected returns of a protective put. The estimation of expected returns is important in the design of an investment strategy, since it links to the major performance measures. Secondly, we suggest a ratcheted