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GMM estimation of spatial autoregressive models with moving average disturbances☆

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ABSTRACT

In this paper, we introduce the one-step generalized method of moments (GMM) estimation methods considered in Lee (2007a) and Liu, Lee, and Bollinger (2010) to spatial models that impose a spatial moving average process for the disturbance term. First, we determine the set of best linear and quadratic moment functions for GMM estimation. Second, we show that the optimal GMM estimator (GMME) formulated from this set is the most efficient estimator within the class of GMMEs formulated from the set of linear and quadratic moment functions. Our analytical results show that the one-step GMME can be more efficient than the quasi maximum likelihood (QMLE), when the disturbance term is simply i.i.d. With an extensive Monte Carlo study, we compare its finite sample properties against the MLE, the QMLE and the estimators suggested in Fingleton (2008a).

1. Introduction

Spatial econometrics models that have a long history in regional, urban and public economics have recently found many applications in macro growth models. These models enable researchers to incorporate spatial dependence among observations into economic analysis. Spatial dependence is a special form of cross-sectional dependence that is determined by locations of observations in space, and is often incorporated into regression specifications in three ways: (i) spatial lag (SAR) model, (ii) spatial error (SEM) model, and the combination of (i) and (ii). The SAR specification involves an autoregressive (AR) process for the spatial lags of the dependent variable such that the dependent variable at a point in space depends on the dependent variables of the surrounding locations. An equilibrium outcome of a theoretical economic model of interacting spatial units often motivates the SAR specification. The SEM specification incorporates the AR process for the spatial lags of the disturbance term. The spatial dependence may stem from measurement errors that tend to vary systematically over space.

A well-known feature of the AR process is that it allows for a global transmission of shocks through global spillovers that agglomerate from higher order neighbors (Anselin, 1988; LeSage and Pace, 2009). However, the AR specification may not be appropriate, if there is strong evidence towards localized transmission of shocks, i.e., shocks that are not transmitted globally. For example, the findings in the empirical literature about the diffusion of technology indicate that the diffusion is more localized in the sense that the productivity effects of innovations decline with the geographical distance between countries (Bottazzi and Peri, 2003; Keller, 2002). Hence, an alternative specification that allows for a localized transmission of shocks is needed. Haining (1978), Anselin (1988) and recently Hepple (2003), Fingleton (2008a,b) consider a spatial moving average process for the disturbances. Following Baltagi and Liu (2011), we will refer to this model as the spatial moving average (SMA) model. As pointed out by these authors, applied researchers sometimes need to treat the transmission of shocks as a local phenomenon. Alternatively, researchers may just be interested in local effects arising from immediate neighbors.

Anselin and Bera (1998) suggest a spatial regression specification (i.e., SARMA) that combines the SMA process for the disturbances and a SAR process for the dependent variable. For example, pollution in a certain location can be modeled as a function not only of the local

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income, but also of the income of the neighbors, and their neighbors' neighbors, and so on. In other words, spillovers travel throughout the whole system and are not limited to the immediate neighbors. Contrarily, the unobserved factors that are affecting pollution in that location are bounded to a small local neighborhood. Behrens et al. (2012) use a SARMA(1,1) specification to model bilateral trade flows between regions based on a dual version of the gravity equation. This specification allows to take into account the interdependence between trade flows through a spatial autoregressive process for the trade flows. Hence, the trade flows from a region to a destination region depend on all the trade flows from the other regions to the destination region. Behrens et al. (2012) consider a spatial moving average process for the disturbance term to model the cross-sectional correlation among disturbance terms.[1] The combination of the spatial autoregressive process for the trade flows and the spatial moving average process for the unobserved factors induces a complicated pattern for the transmission of a region specific shock to other regions. In comparison with a SARAR(1,1) specification, the effect of a region specific shock is heavily concentrated on the immediate neighbors in a SARMA(1,1) specification.

The spatial econometrics literature has mainly focused on the estimators proposed for the estimation of the models that assume a SAR process for the spatial dependence in the disturbance term (Das et al., 2003; Kelejian and Prucha, 1998, 1999, 2010; Lee, 2003, 2004, 2007a, b; Liu et al., 2010). The maximum likelihood (ML) method has received the most attention (Anselin, 1988; LeSage and Pace, 2009). However, the large sample properties of the maximum likelihood estimator (MLE) and the quasi MLE (QMLE) have recently been established by Lee (2004) only for models with spatial AR dependence. The ML estimation can involve a significant computational difficulty for certain weight matrices, when the sample size is large.[2] On the other hand, the generalized method of moments (GMM) and instrumental variable (IV) methods are proven to be computationally more feasible than the ML method. Various two stage least squares estimators (2SLSE) corresponding to the different sets of instrumental variables have been proposed by Anselin (1988), Kelejian and Prucha (1998, 1999, 2007, 2010), and Lee (2003, 2007a). The structure of spatial regression specification determines the possible instruments, which are often constructed from exogenous variables and spatial weight matrices.

Kelejian and Prucha (1998, 1999, 2010) propose a multi-step estimator (GS2SLSE) that involves a combination of the IV and the GMM methods for the spatial model that has a spatial autoregressive process in the dependent variable and the disturbances (SARAR(1,1)). First, an initial estimate of the parameters of the exogenous variables and the autoregressive parameter of the spatial lag of the dependent variable are estimated by the 2SLSE. Then, the residuals from the first step are used to estimate the autoregressive parameter of the spatial lag of the disturbance term by the GMME formulated from a combination of a set of quadratic moment functions. In the final step, the parameters are estimated by the 2SLSE, after transforming the model via a Cochrane–Orcut type transformation to account for the spatial correlation. However, the estimation approach in Kelejian and Prucha (1998) is inefficient relative to the ML method (Prucha, 2012). To increase the efficiency, Lee (2007a,b), Liu et al. (2010), and Lee and Liu (2010) suggest one-step GMM estimators involving sets of moment functions that are linear and quadratic in disturbance terms. The linear moment functions are based on the deterministic part of the spatial lag term and the quadratic moment functions are constructed for exploiting the stochastic part of the spatial lag variable (i.e., the endogenous variable). The quadratic moment functions are chosen in such a way that the resulting one-step GMME can be asymptotically equivalent to the MLE, when disturbances are i.i.d. normal.

The spatial moving average model introduces a different interaction structure. Therefore, it is of interest to investigate the implications of a moving average process for estimation and testing issues. Recently, Fingleton (2008a, 2008b) extends the GMM methodology of Kelejian and Prucha (1998) to a SARMA(1,1) specification. Although the finite sample properties of GS2SLSE are explored in detail for the SARMA(1,1) model, the asymptotic properties are not provided. (Baltagi and Liu, 2011) introduce the estimation approach of Kelejian and Prucha (1998, 1999) to the case of SARMA(0,1) specification in the light of the improvement suggested by Arnold and Wied (2010). The asymptotic distribution of the estimator of the spatial moving average parameter is not provided in both Fingleton (2008a) and Baltagi and Liu (2011). Recently, Kelejian and Prucha (2010) and Drukker et al. (2013) provide a basic theorem regarding the asymptotic distribution of their estimator under fairly general conditions. The estimation approach suggested in Kelejian and Prucha (2010) and Drukker et al. (2013) is characterized as a two-step GMM estimator and can easily be adapted for the estimation of the SARMA(1,1) and SARMA(0,1) models. Finally, although the Kelejian and Prucha approach in Fingleton (2008a) and Baltagi and Liu (2011) has computational advantage, it may be inefficient relative to the ML method.[3]

In this study, we extend the one-step GMM methodology proposed by Lee (2007a) and Liu et al. (2010) to the spatial models that have a moving average process in the disturbance term. For the SARMA(0,1) and SARMA(1,1) specifications, we consider the class of optimal GMM estimators that are formulated from a set of moment functions involving both linear and quadratic moment functions. The best GMME (BGMME) within this class is the one that has the highest asymptotic efficiency. We determine the set of the moment functions for both SARMA(0,1) and SARMA(1,1) specifications that leads to the most efficient GMME, i.e., the BGMME. Along the same line of arguments in Breusch et al. (1999), we show that this set of moment functions is the best one in the sense that any other moment function that can be added to this set does not increase the asymptotic efficiency. Finally, through a Monte Carlo study, the finite sample properties of the BGMME are compared with the MLE, the QMLE and the GS2SLSE. Also, we replicate the empirical results of Behrens et al. (2012) for the SARMA(1,1) specification to evaluate the performance of estimators in an applied research.

This paper is organized as follows. Section 2 elaborates further on the SAR and the SMA disturbance processes. Section 3 presents the model assumptions and their implications. Sections 4 and 5 discuss the GMM estimation of the SARMA(0,1) and the SARMA(1,1) specifications and propose the best moment functions along with the large sample properties of the GMME. The Monte Carlo study and the empirical illustration are carried out in Sections 6 and 7, respectively. Finally, there are concluding remarks. All technical results and derivations are collected in a web appendix available online.

2. Spatial dependence specifications for the disturbance term

In the literature, three parametric specifications have been proposed to model the spatial dependence in the disturbance term: (i) the spatial autoregressive process (SAR), (ii) the spatial moving average process (SMA), and (iii) the spatial error components (SEC) process. In this section, we briefly show the implications of these specifications in terms of

[1] For details, see Section 7.

[2] The likelihood involves the determinant of a matrix, whose dimensions depend on the sample size. For further information, see Das et al. (2003), Kelejian and Prucha (1998, 2010).

[3] Fingleton (2008a) and Baltagi and Liu (2011) do not compare the finite sample efficiency of their estimators with the MLE.