The Quarterly Review of Economics and Finance 53 (2013) 175–187

Contents lists available at SciVerse ScienceDirect

The Quarterly Review of Economics and Finance

journal homepage: www.elsevier.com/locate/qref

Orthogonalized factors and systematic risk decomposition

Rudolf F. Klein[a,*], Victor K. Chow[b,1]

[a] Sweet Briar College, Department of Economics, 134 Chapel Road, Sweet Briar, VA 24595, United States
[b] West Virginia University, Division of Economics and Finance, P.O. Box 6025, Morgantown, WV 26506, United States

ARTICLE INFO

Article history:
Received 19 June 2010
Received in revised form 26 January 2013
Accepted 19 February 2013
Available online 7 March 2013

JEL classification:
G11
G12
G14

Keywords:
Orthogonalization
Systematic risk
Decomposition
Fama-French Model
Asset pricing

ABSTRACT

In the context of linear multi-factor models, this study proposes an egalitarian, optimal and unique procedure to find orthogonalized factors, which also facilitates the decomposition of the coefficient of determination. Importantly, the new risk factors may diverge significantly from the original ones. The decomposition of risk allows one to explicitly examine the impact of individual factors on the return variation of risky assets, which provides discriminative power for factor selection. The procedure is experimentally robust even for small samples. Empirically we find that even though, on average, approximately eighty (sixty-five) percent of style (industry) portfolios' volatility is explained by the *market* and *size* factors, other factors such as *value*, *momentum* and *contrarian* still play an important role for certain portfolios. The components of systematic risk, while dynamic over time, generally exhibit negative correlation between *market*, on one side, and *size*, *value*, *momentum* and *contrarian*, on the other side.

1. Introduction

Under the traditional single-factor Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM), the market beta captures a stock's systematic risk for all rational, risk-averse investors. Therefore, a decomposition of the market beta is sufficient to break down the systematic risk of a stock.[2] For example, Campbell and Vuolteenaho (2004) break the market beta of a stock into a 'bad' component, that reflects news about the market's future cash flows, and a 'good' component, that reflects news about the market's discount rates. In an earlier paper, Campbell and Mei (1993) show that the market beta can be decomposed into three sub-betas that reflect news about future cash flows, future real interest rates and a stock's future excess returns, respectively. Acharya and Pedersen (2005) develop a CAPM with liquidity risk by dividing the market beta of a stock into four sub-betas that reflect the impact of illiquidity costs on the systematic risk of an asset. Researchers frequently apply decompositions of the market beta to examine the size and/or book-to-market anomalies. Although beta-decompositions are useful to describe the structure and source of systematic variation of returns on risky assets, they are complicated under multi-factor frameworks. For instance, Campbell and Mei (1993) show that one complication is due to the possible covariance between the risk price of one factor and the other factors, which prevents identifying a neat linear relationship between the overall beta of an asset and its beta of news about future cash flows.

The purpose of this paper is to develop an optimal procedure to identify the underlying uncorrelated components of common factors, by a simultaneous and symmetric orthogonal transformation of sample data, such that the linear dependence is removed and the systematic variation of stock returns becomes decomposable. We empirically compare our approach with two popular orthogonalization methods, Principal Component Analysis (PCA) and the Gram-Schmidt (GS) process, and unsurprisingly find that our technique has the essential advantage of maintaining maximum resemblance with the original factors.[3]

* Corresponding author. Tel.: +1 434 321 4773.
E-mail addresses: rklein@sbc.edu (R.F. Klein), victor.chow@mail.wvu.edu (V.K. Chow).
[1] Tel.: +1 304 293 7888.

[2] According to the CAPM, the systematic risk is measured as $(\beta_j \sigma_{RM})^2$. Since the market factor is the only priced risk factor faced by all investors, β_j is sufficient to determine the systematic risk.

[3] For instance, Baker and Wurgler (2006, 2007) employ PCA to develop measures of investor sentiment, shown to have significant effects on the cross-section of stock

1062-9769/$ – see front matter
http://dx.doi.org/10.1016/j.qref.2013.02.003

In the past two decades, one of the most extensively researched areas in finance has concentrated on alternative common risk factors, in addition to market risk, that could characterize the cross-section of expected stock returns. Fama and French (1992, 1993, 1996, 1998) document that a company's market capitalization, *size*, and the company's *value*, which is assessed by ratios of book-to-market (B/M), earnings to price (E/P) or cash flows to price (C/P), together predict the return on a portfolio of stocks with much higher accuracy than the market beta alone, or the traditional CAPM.[4] In addition to the *size* and *value* effects, Jegadeesh and Titman (1993), Jegadeesh and Titman (2001), Rouwenhorst (1998), and Chan, Jegadeesh, and Lakonishok (1996) report that short-term past returns or past earnings predict future returns. Average returns on the best prior performing stocks (i.e., the *winners*) exceed those of the worst prior performing stocks (i.e., the *losers*), attesting the existence of *momentum* in stock prices. Conversely, De Bondt and Thaler (1985, 1987) detect a *contrarian* effect by which stocks exhibiting low long-term past returns outperform stocks with high long-term past returns. De Bondt and Thaler (1985, 1987), Chopra, Lakonishok, and Ritter (1992), and Balvers, Wu, and Gilliland (2000) suggest a profitable contrarian strategy of buying the *losers* and shorting the *winners*.

Consequently, for the determination of the return generating process for risky assets, one needs to consider more than just the market risk factor. For this reason, multi-factor market models have been widely employed by both academics and practitioners. Under the multi-factor framework, the expected excess return on a risky asset is specified as a linear combination of beta coefficients and expected premia of individual factors. Fama and French (1993) emphasize that, if there are multiple common factors in stock returns, they must be in the market return, as well as in other well-diversified portfolios that contain these stocks. This indicates that returns on common factors must be, to some degree, correlated with the market and with each other. Consequently, in a multiple linear regression setting, although the beta coefficient corresponding to an individual factor provides a sensitivity measure of an asset's return to the factor's variation, it is not sufficient to assess the systematic variation of the asset's return with respect to that factor. The volatility of an asset's return is determined jointly not only by the betas, but also by the variances and covariances of the factors' premia. Therefore, determining the factors' underlying uncorrelated components helps us achieve a clearer identification of the separate roles of common factors in stock returns.

This paper proposes an optimal simultaneous orthogonal transformation of factor returns. The data transformation allows us to identify the underlying uncorrelated components of common factors. Specifically, the inherent components of factors retain their variances, but their cross-sectional covariances are equal to zero. Moreover, a multi-factor regression using the orthogonalized factors has the same coefficient of determination, *R*-square, as that using the original, non-orthogonalized factors. Importantly, the coefficient of determination (the ratio of systematic variation to the overall volatility of a risky asset) is a measure of the systematic risk of an asset. Therefore, disentangling the *R*-square based on factors' volatilities and their corresponding betas enables us to decompose the systematic risk. For that, we need to extract the core, standalone components of common factors. Fama and French (1993) clearly demonstrate that since the market return is a mixture of the multiple common factors, an orthogonalization of the market factor is necessary so that it can capture common variation in returns left from other factors such as *size* or *value*. We argue that not only the market factor, but all factors need to be orthogonally transformed to eliminate any dependence among them. Although Fama and French's (1993) orthogonalization procedure for the market factor is straightforward, it cannot be extended to eliminate the correlations between all variables in a model, without generating two related biases. Firstly, similarly to GS, it leaves one factor (call it *leader*) unchanged. Secondly, it is a sequential (i.e., order-dependent) procedure. Therefore, a different selection of the leader or a different orthogonalization sequence generates different transformation results. Our method avoids these two biases by construction.

Using Monte Carlo simulations, we demonstrate that our orthogonal transformation is robust, in that it produces precise estimates of the population systematic risk even for small samples. By applying our methodology to some of the Kenneth French's style and industry portfolios, we show empirically that the systematic return variation can now be unequivocally allocated to the common factors.[5] We find that, over a time period from January 1931 to December 2008, the *market* and *size* factors are the largest sources of systematic risk, while other factors such as *value*, *momentum* and *contrarian* play relatively small roles in stock volatility.

The paper is organized as follows. In Section 2, after explaining why the systematic risk decomposition is problematic under multi-factor models, we present our procedure of symmetric orthogonal transformation and risk decomposition. In Section 3, we illustrate the procedure empirically, using monthly U.S. Research Returns Data obtained from Kenneth French's Data Library, for the time interval January 1931–December 2008. The final section of the paper provides concluding remarks.

2. Orthogonalization procedure

Suppose a risky asset *j*'s return generating process is linearly determined by a set of *K* common factors (f^k), such as *market* (*RM*), *size* (*SMB*), *value* (*HML*), *momentum* (*Mom*), and *long-term reversal* (*Rev*), as shown in the following general linear factor model.

$$r_t^j = \alpha_j + \sum_{k=1}^{K} \beta_{k_j} f_t^k + \varepsilon_t^j, \tag{1}$$

where f^k are assumed to be uncorrelated with the residual term (ε_j), but not with each other. For instance, the *market* factor is a mixture of the multiple common factors, while the factor-mimicking portfolios of *size*, *value*, *momentum* and *contrarian* are all formed using securities in the same market, and thus their returns are not uncorrelated.

The systematic return variation (σ_s^2) of asset *j* can then be measured as

$$\sigma_{s_j}^2 = \sum_{l=1}^{K} \sum_{k=1}^{K} \beta_{k_j} \beta_{l_j} Cov(f^k, f^l), \tag{2}$$

while the coefficient of determination, *R*-square, is the ratio of systematic variation to total return variation ($\sigma_{s_j}^2/\sigma_j^2$).

It is important to note that under the multi-factor framework, systematic risk depends not only on the beta coefficients but also on the factors' variance–covariance. Thus, beta coefficients alone are

returns. Boubakri and Ghouma (2010) remove the multicollinearity between their variables using the Gram-Schmidt algorithm.

[4] Fama and French (1992, 1996, 1998) show that the investment strategy of buying the *Small* – *Value* stocks and shorting the *Big* – *Growth* stocks produces positive returns.

[5] Kenneth French's Data Library is located at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.