Research in Economics 68 (2014) 306–314

Contents lists available at ScienceDirect

Research in Economics

journal homepage: www.elsevier.com/locate/rie

Coordination, timing and common knowledge ☆

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ARTICLE INFO

Article history:
Received 21 April 2014
Accepted 25 April 2014
Available online 16 May 2014

Keywords:
Coordination
Common knowledge
Global game
Timing frictions
Calvo frictions

ABSTRACT

There is tight link between coordination and common knowledge. The role of higher order beliefs in static incomplete information games has been widely studied. In particular, information frictions break down common knowledge. A large body of literature in economics examine dynamic coordination problems when there are timing frictions, in the sense that players do not all move at once. Timing frictions in dynamic coordination games play a role that is closely analogous to information frictions in static coordination games.

This paper makes explicit the role of higher order beliefs about timing in dynamic coordination games with timing frictions. An event is said to be effectively known if a player knew the event when he last had an option to change his behavior. The lack of effective common knowledge of the time drives results of dynamic coordination games.

1. Introduction

We sometimes choose to do things only because other people are doing them. David Hume referred to the behavior chosen in such situations as *conventions*. A leading example of a convention for Hume and philosophers following him was language. Since everyone else uses the word “cat” to refer to a cat and the word “dog” to refer to a dog, I also do so, but if everyone was using “cat” to refer to a dog and “dog” to refer to a cat, then I would surely have an incentive to switch to the latter usage. Modern game theory describes such situations as *coordination games* and the existence of multiple stable conventions as multiple equilibria.

Coordination problems are ubiquitous in many important social contexts. The importance of incentives to coordinate, or strategic complementarities, has been highlighted in recent years by the global financial crisis. Lenders from the shadow banking sector had an incentive to lend less if others were lending less (via various economic channels), giving rise to strategic complementarities, or coordination incentives, that exacerbated the market freeze. Holders of European sovereign debt had an incentive to hold less debt if others were holding less, since this would drive up interest rates and thus increase the likelihood of default.

Foundational analysis of coordination games has long noted the importance of “higher order beliefs” in understanding coordination. Higher order beliefs refer to each individual's beliefs and knowledge not only about directly payoff relevant events, but also about the beliefs and knowledge of others, their beliefs about others' beliefs, and so on. The philosopher

☆ The paper was prepared for the June 2103 Ca' Foscari University of Venice International Workshop in Economic Theory on “Where Do We Stand”? I am grateful to Yoram Moses for a conversation where he first explained the relation between timing and common knowledge to me, and for comments on this paper. This conversation led me to write a 1995 unpublished paper on “Co-operation and Timing” (Morris, 1995); this paper encompasses and generalizes that working paper.

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http://dx.doi.org/10.1016/j.rie.2014.04.004
1090-9443/

David Lewis emphasized in Lewis (1969) that for language to work, it is necessary not only for participants in a conversation to share vocabulary, so that we all use "cat" to refer to cat, etc., but it is also necessary for all of us to know that we all use "cat" to refer to cat, and so on. Lewis used the expression "common knowledge" to refer to such infinite chains of iterated knowledge. When Aumann (1976) introduced the formal study of common knowledge into economics, he credited Lewis with introducing this formulation. A large body of literature has grown up showing how coordination can be tightly related to relevant higher order beliefs. While common knowledge may be necessary for perfect coordination, as suggested by Lewis, one general insight is that "approximate common knowledge" generally suffices (Monderer and Samet, 1989). Specifically, say that an event is p-believed if at least proportion p believe it with probability at least p. Say that an event is "common p-belief" if it is p-believed, it is p-believed that it is p-believed, and so on.[1] For any given coordination game, behavior that would be an equilibrium if there was common knowledge of the payoffs of that game will still be an equilibrium if there is only common p-belief as long as p is close enough to one. How close p needs to be to 1 depends on how strong the incentives are in the coordination game. This observation highlights that public or almost public events play a key role in coordinating behavior. We also know that approximate common knowledge in this sense is a stringent requirement (Rubinstein, 1989; Weinstein and Yildiz, 2007). In the "global games" literature (Carlsson and van Damme, 1993), a small amount of noise about payoffs implies a dramatic failure of approximate common knowledge, and leads to a prediction that strategically safe behavior will be played; thus in symmetric binary action coordination games, each player will end up playing the "Laplacian" equilibrium corresponding to each player choosing a best response to a diffuse (i.e., uniform) belief over the proportion of his opponents who will choose each action(Morris and Shin, 2003).[2]

But timing is also important for coordination. I am often concerned not only to take the same action as others, but also to take it at the same time. This suggests that higher order beliefs about timing should also matter in coordination problems. It is often argued that events which are particularly "public", and thus approximate common knowledge, are important in coordinating a switch from one equilibrium to another. Thus – to stick to topical examples – Blanchard (2013) and others have highlighted the importance of the public announcement of the European Central Bank's outright monetary transaction program in triggering a switch to a "new equilibrium" in the European sovereign debt crisis. Romer (2013) argues about the importance of public communication in generating a shift of equilibria both for Roosevelt at the start of the New Deal and for Abe in the Japanese monetary policy in 2013. Fearon (2011) argues that revolutions often follow elections that have been stolen by a dictator, not because the theft reveals any new information (it was commonly known that the election would be stolen), but because revolutions are extreme coordination problems (no one wants to be a part of a small minority who revolt) and the stealing of the election provides the public event to coordinate the timing of the revolution.

But while the higher order belief foundations of coordination in static games have been much studied, the higher order belief foundations of coordination and timing have not (at least in economics). In particular, there are large and important bodies of literature, which will be reviewed in Section 3, that introduce timing frictions into economic models. The effect of timing frictions is to make it harder to coordinate behavior, because they break down approximate common knowledge of time. The main purpose of this paper is to present such higher order belief foundations.

To understand the issues, it is useful to consider a common knowledge "paradox". Many entertaining common knowledge paradoxes, concerning, for example, cheating spouses, hats and railway carriages (Geanakoplos, 1992; Fagin et al., 1995), have been used to illustrate the logical importance of higher order beliefs. But the following paradox concerns timing.[3] There is a continuum of individuals whose clocks are not perfectly synchronized. In particular, they are slow by an amount between 0 and 4 min relative to the "true time", with the delay uniformly distributed in the population. Each individual does not know how slow his clock is and has a uniform belief about the delay. At what time does it become common knowledge that the true time is, say, 8:00 a.m. or later? The answer is never. Only when the true time reaches 8:04 does everyone know that the true time has reached 8:00. Only at 8:08 does everyone know that everyone knows that it is past 8:00, and so on. Thus it never becomes common knowledge.

But the paradox gets worse. When does it become common $3/4-$belief – in the sense described above – that the true time has reached 8:00? An individual only assigns probability 3/4 to the true time being after 8:00 when his own clock reaches 7:59. At this point, the true time is after 8:00 as long as his clock is delayed by at least 1 min, a 3/4 probability event. It is not until a true time of 8:02 that proportion 3/4 of individuals observe a time after 7:59: this is because at true time 8:02, individual clock times are uniformly distributed between 7:58 and 8:02. Thus only at 8:02 is it $3/4-$believed – i.e., 3/4 of the population assign probability at least 3/4 – that the true time is after 8:00. It is only at 8:04 that it is $3/4-$believed that it is $3/4-$believed that the time is after 8:00, and so on. So it is also never common $3/4-$belief that the time is after 8:00. We will formalize and generalize this argument and verify that for *any* $p > 1/2$ it is *never* common p-belief that the true time is after 8:00. But, for any $p \leq 1/2$, there is common p-belief that the true time is after 8:00 from 8:00 on.

[1] This definition is a variant of the definition in Monderer and Samet (1989): their definition corresponds to replacing "at least proportion p" with "everyone". The modified definition is relevant for analysis in this paper.

[2] The selection of the Laplacian equilibrium is an implication of the more general result that potential maximizing equilibria are selected in global games if the underlying game has a potential.

[3] This paradox, together with a formal analysis of knowledge and common knowledge about timing, appears in Section 8 of Halpern and Moses (1990). A more entertaining cheating spouse version of this paradox appears in the Stanford Ph.D. thesis, Moses (1986), of Yoram Moses. The thesis also discusses atemporal and temporal versions of approximate common knowledge.