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Refraction of bulk spin waves on a boundary of two homogeneous easy-axis antiferromagnetic media

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Abstract

Spin-wave refraction index is defined on the boundary of two bulk homogeneous antiferromagnetic media having different parameters of exchange coupling and anisotropy. The effect of spin-wave birefringence is revealed in such structure. The expression for reflection coefficient is obtained on the boundary. The dependencies of the reflection coefficient on frequency and value of the external homogeneous permanent magnetic field are obtained for both branches of spin wave spectrum.

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Let us consider the structure consisting of two bulk semi-infinite homogeneous antiferromagnetic media having different values of the parameters of exchange coupling α, α', δ and anisotropy β, β' [1]. Easy axis and external permanent homogeneous magnetic field are directed along Oz axis. The yOz plane is the plane of the contact of these media. The energy density of each of two parts, marked with (j) index, is given with the assumption of the condition $\mathbf{M}_p^{(j)^2}(\mathbf{r},t) = \text{const}$ ($p = 1,2$ correspond to two sublattices) by

$$w^{(j)} = \sum_{k=1,2}\left[\frac{\alpha^{(j)}}{2}\left(\frac{\partial \mathbf{m}_1^{(j)}}{\partial x_k} + \frac{\partial \mathbf{m}_2^{(j)}}{\partial x_k}\right)^2 + \frac{\alpha'^{(j)}}{2}\left(\frac{\partial \mathbf{m}_1^{(j)}}{\partial x_k}\cdot\frac{\partial \mathbf{m}_2^{(j)}}{\partial x_k}\right)\right] + \frac{(\beta^{(j)} - \beta'^{(j)})}{2}(m_{1x}^{(j)^2} + m_{1y}^{(j)^2} + m_{2x}^{(j)^2} + m_{2y}^{(j)^2}) - H_0(M_{1z}^{(j)} + M_{2z}^{(j)}) + \delta^{(j)}\mathbf{M}_1^{(j)}\mathbf{M}_2^{(j)}. \tag{1}$$

Here, $\mathbf{M}_p^{(j)} = (-1)^{p+1}M_0\mathbf{e}_z + \mathbf{m}_p^{(j)}$, M_0 is saturation magnetization of sublattices, $\mathbf{m}_p^{(j)}$ are the vectors characterizing the small deviations of magnetic moment from the ground state. We shall use the formalism of spin density [2], according to which the magnetic moment can be written as

$$\mathbf{M}_p^{(j)}(\mathbf{r},t) = M_0\Psi_p^{(j)^+}(\mathbf{r},t)\boldsymbol{\sigma}\Psi_p^{(j)}(\mathbf{r},t). \tag{2}$$

Here, $\Psi_p^{(j)}$ are quasiclassical wave functions playing the role of the order parameter of spin density, σ are Pauli matrices.

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Lagrange equations for $\Psi_p^{(j)}$ are given by

$$i\hbar \frac{\partial \Psi_p^{(j)}(\mathbf{r},t)}{\partial t} = -\mu_0 \mathbf{H}_{e_p}^{(j)}(\mathbf{r},t)\boldsymbol{\sigma}\Psi_p^{(j)}(\mathbf{r},t), \quad (3)$$

where μ_0 is Bohr magneton,

$$\mathbf{H}_{e_p}^{(j)} = -\frac{\partial w^{(j)}}{\partial \mathbf{M}_p^{(j)}} + \frac{\partial}{\partial x_k}\frac{\partial w^{(j)}}{\partial(\partial \mathbf{M}_p^{(j)}/\partial x_k)}.$$

The material is magnetised to be parallel to $\mathbf{e}_z$ in the ground state. Using the linear perturbation theory, we shall write the solution of Eq. (3) as

$$\Psi_1^{(j)}(\mathbf{r},t) = \exp(i\mu_0 H_0 t/\hbar)\cdot \begin{pmatrix} 1 \\ \chi^{(j)}(\mathbf{r},t)\end{pmatrix},$$
$$\Psi_2^{(j)}(\mathbf{r},t) = \exp(-i\mu_0 H_0 t/\hbar)\cdot \begin{pmatrix} \xi^{(j)}(\mathbf{r},t) \\ 1\end{pmatrix}. \quad (4)$$

Here, the upper element in $\Psi_1^{(j)}$ and lower one in $\Psi_2^{(j)}$ correspond to the ground state, whereas $\chi^{(j)}(\mathbf{r},t)$ and $\xi^{(j)}(\mathbf{r},t)$ are the small additions characterizing deviations of the magnetic moments of sublattices from the ground state. Linearizing Eq. (3) and taking into consideration Eq. (4), we obtain

$$-\frac{i\hbar}{2\mu_0 M_0}\frac{\partial \chi^{(j)}(\mathbf{r},t)}{\partial t} = \{\alpha'^{(j)}\Delta - \delta^{(j)}\}\xi^{(j)^*}(\mathbf{r},t) + \{\alpha^{(j)}\Delta - [\beta^{(j)} - \beta'^{(j)}] - \tilde{H}_0 - \delta^{(j)}\}\chi^{(j)}(\mathbf{r},t),$$

$$-\frac{i\hbar}{2\mu_0 M_0}\frac{\partial \xi^{(j)}(\mathbf{r},t)}{\partial t} = \{\alpha'^{(j)}\Delta - \delta^{(j)}\}\chi^{(j)^*}(\mathbf{r},t) + \{\alpha^{(j)}\Delta - [\beta^{(j)} - \beta'^{(j)}] - \delta^{(j)} + \tilde{H}_0\}\xi^{(j)}(\mathbf{r},t). \quad (5)$$

Here, $\tilde{H}_0 = H_0/M_0$. Performing the time and spatial Fourier transformations, we obtain the dispersion law

$$\Omega^{\pm} = \{[(\alpha^{(j)} - \alpha'^{(j)})k^{(j)^2} + \beta^{(j)} - \beta'^{(j)}] \times [(\alpha^{(j)} + \alpha'^{(j)})k^{(j)^2} + \beta^{(j)} - \beta'^{(j)} + 2\delta^{(j)}]\}^{1/2} \pm \tilde{H}_0. \quad (6)$$

Here, $\Omega = \omega\hbar/2\mu_0 M_0$. It should be noted that this expression contains not only parameters, which dominate at the propagation of long-wave excitations (see, for instance, Ref. [1]), but all the rest of the items too. Their contribution becomes essential as for the propagation of short waves, as in the case of the media having weak enough exchange interaction between sublattices. This fact is especially important on account of the presence of wide opportunities of the creation of new artificial materials having unique properties.

Defining the refraction index as the ratio of wave numbers in different parts of the structure, we obtain $(n^{\pm\pm})^2 = (k_2^{\pm\pm})^{(2)}/(k_1^{\pm\pm})^2$.

Here, as it follows from Eq. (6),

$$(k_j^{\pm\pm})^2 = \frac{\delta^{(j)}}{\alpha^{(j)} + \alpha'^{(j)}}\cdot\Bigg\{\pm\,\mathrm{sgn}(\alpha^{(j)} - \alpha'^{(j)}) \times \Bigg[1 - \frac{\alpha^{(j)} + 2\alpha'^{(j)}}{\alpha^{(j)} - \alpha'^{(j)}}\cdot\frac{\beta^{(j)} - \beta'^{(j)}}{\delta^{(j)}} + \frac{\alpha^{(j)} + \alpha'^{(j)}}{\alpha^{(j)} - \alpha'^{(j)}}\times\left(\frac{\Omega \pm \tilde{H}_0}{\delta^{(j)}}\right)^2 - \frac{3\alpha^{(j)^2} - 4\alpha'^{(j)^2}}{4(\alpha^{(j)} - \alpha'^{(j)})^2}\cdot\left(\frac{\beta^{(j)} - \beta'^{(j)}}{\delta^{(j)}}\right)^2\Bigg]^{1/2} - 1 - \frac{\alpha^{(j)}}{2(\alpha^{(j)} - \alpha'^{(j)})}\cdot\frac{\beta^{(j)} - \beta'^{(j)}}{\delta^{(j)}}\Bigg\}. \quad (7)$$

Here, the first upper index at $k_j^{\pm\pm}$ corresponds to the choice of the sign before $\mathrm{sgn}(\alpha^{(j)} - \alpha'^{(j)})$, and the second one—to that before $\tilde{H}_0$.

Let us use the boundary conditions at the contact place $x = 0$ (prime over $\chi^{(j)}$, $\xi^{(j)}$, $\chi^{*(j)}$, $\xi^{*(j)}$ is derivative on x):

$$\chi^{(1)} = \chi^{(2)},$$
$$\xi^{*(1)} = \xi^{*(2)},$$
$$\alpha^{(1)}\chi^{(1)'} + \alpha'^{(1)}\xi^{*(1)'} = \alpha^{(2)}\chi^{(2)'} - \alpha'^{(2)}\xi^{*(2)'},$$
$$\alpha'^{(1)}\chi^{(1)'} + \alpha^{(1)}\xi^{*(1)'} = \alpha'^{(2)}\chi^{(2)'} - \alpha^{(2)}\xi^{*(2)'}, \quad (8)$$

and four conjugated equations.

Firstly, these linear homogeneous equations lead to the Snell's law for spin waves:

$$\frac{\sin\theta_1}{\sin\theta_2^{\pm\pm}} = \frac{k_2^{\pm\pm}}{k_1^{\pm\pm}} = n^{\pm\pm}.$$

Here, θ_1 is the angle of incidence and $\theta_2^{\pm\pm}$ are the angles of refraction.

Secondly, we can find the reflecting ability of the boundary of two homogeneous antiferromagnetic parts. For this purpose, we shall associate the following functions to incident, reflected and transmitted spin waves (after Fourier transformations on t, y and z):

$$\begin{pmatrix}\chi \\ \xi^* \\ \chi^* \\ \xi\end{pmatrix}_I = \exp(ik_{1x}^{++}x)\begin{pmatrix} A^{(1)++} - B_1^{(1)} \\ -C^{(1)++} \\ 0 \\ 0\end{pmatrix} + \exp(ik_{1x}^{+-}x)\begin{pmatrix} 0 \\ 0 \\ A^{(1)+-} - B_2^{(1)} \\ -C^{(1)+-}\end{pmatrix},$$