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Multiple equilibria in the Aghion–Howitt model

Jess Benhabib *

Department of Economics, New York University, 19 West 4th Street, New York, NY 10003, United States

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ABSTRACT

We show that the simple Aghion–Howitt model exhibits oscillatory indeterminacy in the process of creative destruction as firms undertaking new research do not internalize their effect on existing firms. A simple calibration shows that indeterminacy occurs for quite plausible parametrizations of the share of the intermediate good.

1. Introduction

Endogenous growth models generally model spillovers and external effects that overcome diminishing returns and sustain persistent growth without relying on assumptions of exogenous technical change. Such externalities and spillovers however can generate coordination problems and multiple equilibria. Benhabib and Perli (1994) have shown that this can happen in the classic Lucas (1988) growth model, and Benhabib et al. (1994) have demonstrated that it can happen in the Romer (1990) growth model. Here we show that the classic Aghion and Howitt (1992, 1999) quality ladder model of creative destruction also exhibits oscillatory indeterminacy, although for different reasons. Furthermore very simple calibration indicates that indeterminacy is in fact a likely outcome. These basic results are also closely related to results in Matsuyama (1999) and Deneckere and Judd (1992), though the simple Aghion–Howitt model starkly illustrates the features and forces that generate oscillations and the multiplicity of equilibria.

2. The basic Aghion–Howitt model

We start by sketching a very simple version of the Aghion–Howitt creative destruction model. We abstract from capital and we focus only on production and on research that produces new intermediate goods. The output of the final product y is simply

$$y_t = Ax_t^{\alpha}, \quad \alpha < 1$$

where x_t is the intermediate input. Its price is

$$p_t = A_t\alpha(x_t)^{\alpha-1}$$

because it earns its marginal product. The production of the single intermediate input uses one unit of labor to produce one unit of x with a linear technology $x = L_x$. Therefore, the profits in the intermediate goods production, π_t, are given by

$$\pi_t = p_t x - w_t x = A_t\alpha(x_t)^{\alpha} - w_t x_t$$

* Tel.: +1 212 998 8971; fax: +1 212 995 4186.
E-mail address: jess.benhabib@nyu.edu

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Maximizing profits in this sector, the output of the intermediate goods sector is

$$x_t = Arg\ \max(A_t\alpha(x_t)^\alpha - w_t x_t)$$

so that

$$\alpha^2 A x^{\alpha-1} = w \quad \text{and} \quad x_t = \left(\frac{\alpha^2}{\left(\frac{w_t}{A_t}\right)}\right)^{1/1-\alpha} \equiv x(\omega_t), \quad \text{where} \quad \omega = \frac{w}{A}$$

Then profits can be written as

$$\pi = A_t \alpha x^\alpha - w_t x = (\alpha^{-1}-1) w_t x_t = A_t(\alpha^{-1}-1)\alpha^2 x_t^\alpha$$
$$\pi = A_t(\alpha^{-1}-1)(\alpha)^{2/1-\alpha}(\omega_t)^{\alpha/\alpha-1} \equiv A_t \tilde{\pi}_t(\omega_t)$$

Note that if $\alpha = 1$, $dp/dx = 0$, and there are no profits.

Unlike Matsuyama (1999) or Deneckere and Judd (1992), in our simple model at any time t there is a single intermediate good x_t and a final good consumption good y_t. When a new more efficient intermediate good is invented, it immediately replaces the older less efficient one. The usual Dixit–Stiglitz structure with production complementarities between multiple goods or between a continuum of intermediate goods are absent. Therefore in our simple model there is no incentive to bunch the invention of new intermediate goods in order to take advantage of production complementarities.[1] We will see nevertheless that oscillations in output will still emerge as in Matsuyama (1999) or Deneckere and Judd (1992). Furthermore for plausible parametrizations, there will be a continuum of equilibria.

In the research sector if n persons do research, the probability density function of an invention at time τ is given by the Poisson density,

$$f(\tau) = (\lambda n) e^{-(\lambda n)\tau},$$

so that the probability of an invention by time τ is:

$$F(\tau) = 1 - e^{-(\lambda n)\tau}, \quad F(0) = 0, \quad F(\infty) = 1$$

Total profits, if the firm survives until T when a new invention arrives are given by

$$\int_0^T \pi e^{-rs}\, ds = \frac{\pi}{r}(1 - e^{-rT})$$

The expected value of firm is discounted profits summed over survival probabilities to each T (that is by probabilities that there is an invention exactly at T, given by the density function above). For simplicity we take the rate of interest r at which profits are discounted as given, as in a small open economy. This yields the expected value of the firm:

$$\begin{aligned} V &= \int_0^\infty (\lambda n) e^{-(\lambda n)T} \int_0^T \pi e^{-rs}\, ds\, dT \\ &= \frac{\pi}{r}\left(\int_0^\infty (\lambda n) e^{-(\lambda n)T}(1 - e^{-rT})\, dT\right) \\ &= \frac{\pi}{r}\left(\int_0^\infty (\lambda n) e^{-(\lambda n)T}\, dT - \int_0^\infty (\lambda n) e^{-(r+\lambda n)T}\, dT\right) \\ &= \frac{\pi}{r}\left(1 - \frac{\lambda n}{r+\lambda n}\right) = \frac{\pi}{r}\left(\frac{r}{r+\lambda n}\right) = \frac{\pi}{r+\lambda n} \end{aligned}$$

The present value of profits then is discounted by the rate of interest plus the probability of demise. Therefore, the expected value of a firm satisfies:

$$rV = \pi - (\lambda n)V$$

Total labor is divided between research and production:

$$L = n_t + x_t \tag{1}$$

There is arbitrage across labor markets for research and manufacturing. Assuming $A_{t+1} = \gamma A_t$, because improvements on the technology ladder occur with a factor γ, and t is the time between innovations, the optimal choice of n_t is given by:

$$Max_{n_t}\ n_t \lambda V_{t+1} - w_t n_t$$

[1] See Matsuyama (1999) or Shleifer (1986).