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Magnetodeformational effect in ferrogel objects

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Abstract

Deformation arising in a ferrogel sample in response to applied uniform magnetic field is investigated assuming that ferrogel is an isotropic linearly magnetizable medium. For small deformations the results on solid and hollow spherical samples are presented. We find that compliance of a hollow sphere (vesicule) to elongation grows with diminution of the wall thickness; this is accompanied by the decrease of the internal volume of the vesicule.

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Ferrogels make a challenging class of soft magnetic matter [1–3]. These materials are distinguished by low-values of their elastic moduli (10^4 Pa and less) so that specific deformations induced by moderate magnetic fields reach from tens to hundreds percent. This smart behavior ensures for ferrogels wide technological prospects.

Designing a process or device employing such media, one needs to predict the field-induced deformation of a particular ferrogel sample. This means solving a coupled elastic/magnetostatic problem. By now only a few simple cases have been examined. Our work presents the results of evaluation of the magnetodeformational effect (MDE) in solid and hollow spheroidal ferrogel bodies, where the constituting material is modeled by a continuum obeying the Hook elasticity and the Langevin magnetization laws.

In the very first attempts on MDE [4,5] exclusively the case of a solid (non-hollow) spherical body was studied. Moreover, an essential assumption was that a sample, being a sphere at $\boldsymbol{H}=0$, on application of a uniform field stretches into a spheroid. Later on an exact solution was found [6], which established that under field a sphere assumes a shape that is axisymmetrical about $\boldsymbol{H}$ but not a spheroidal one. The contour of the cross-section of this body is rendered by a cubic equation. In the parametric form in the (ρ,z) plane of a cylindrical coordinate framework $(\boldsymbol{H}||Oz)$ it writes

$$\rho = \sqrt{(2-2P_2)/3}[1+\alpha(4P_2-3)], \qquad (1)$$

$$z = \sqrt{(1+2P_2)/3}[1+\alpha(4P_2+6)],$$

where $P_2(\cos\vartheta)$ is the second Legendre polynomial of the meridional angle, and α is a parameter proportional to the squared magnetization so that at the initial state where $\alpha=0$ Eq. (1) unite into $\rho^2+z^2=1$.

The elongation effect is quadratic in the field strength, and so we define the initial MD susceptibility of a sphere as

$$\kappa = \frac{G}{\chi^2}\frac{d\varepsilon}{d(H^{(i)})^2} = G\frac{d\varepsilon}{dM^2}\bigg|_{H\to 0} = \begin{cases} 4\pi/15\,(\text{approx.}), \\ 20\pi/57\,(\text{exact}). \end{cases} \qquad (2)$$

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Here ε, see also definition (7) below, is the elongation of the body scaled with the outer radius of the sphere and G is the elastic modulus. Product of the initial magnetic susceptibility χ and the magnetic field inside the body $H^{(i)}$ equals its initial magnetization M. As Eq. (1) shows, the exact result exceeds the approximate one by about 30%.

Recently the approach, in its approximate and exact forms, was extended [7] to the situation, where the initial state $H = 0$ of a ferrogel body is an arbitrary ellipsoid of revolution. The configurations assumed under field are calculated numerically and shown to never be spheroidal.

An interesting new problem occurs when a hollow ferrogel body is considered. For us it was inspired by Ref. [7], where self-assembling of shells consisting of polymers with embedded ferrite nanoparticles is reported. Accordingly, for our model study we take a sphere with an empty concentric spherical cavity, see Fig. 1. Medium (1) is the carrier fluid, object (2) is a hollow sphere (vesicule) made of a homogeneous linearly magnetizable ferrogel with Hookean elasticity, and (3) is the carrier fluid, which occupies the cavity. For definiteness, we take that in the vesicule wall there virtually exists a thin channel so that the cavity may exchange its content with the outer fluid surrounding it. Therefore, conservation of the internal volume of the vesicule is not required.

Denoting the applied uniform field as H_0 and introducing the induced one in the usual way as

$$H = H_0 - \nabla\psi, \quad \Delta\psi = 0, \tag{3}$$

and assuming the linear magnetization law $\boldsymbol{M} = \chi \boldsymbol{H}$ for the ferrogel one gets the boundary conditions for Eq. (3):

$$(1+4\pi\chi)\frac{\partial\psi^{(2)}}{\partial n} - \frac{\partial\psi^{(1)}}{\partial n} = 4\pi\chi H_0 \cos\vartheta,$$
$$\psi^{(2)} = \psi^{(1)}; \tag{4}$$

$$(1+4\pi\chi)\frac{\partial\psi^{(2)}}{\partial n} - \frac{\partial\psi^{(3)}}{\partial n} = 4\pi\chi H_0 \cos\vartheta,$$
$$\psi^{(2)} = \psi^{(3)}.$$

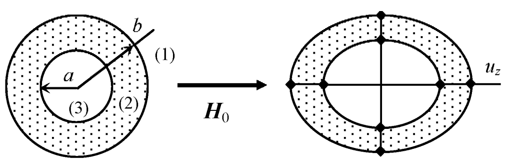

Fig. 1. Schematic representation of a ferrovesicule, i.e., a hollow sphere made of a ferrogel medium and both surrounded and filled with a non-magnetic fluid in the field-free (i.e., initial) and deformed due to a uniform field $\boldsymbol{H}$ states.

The corresponding elasticity equations are

$$\nabla\cdot\mathbf{T} + 1/2\chi\nabla(\mathrm{H}^2) = 0, \quad \mathbf{T} = \lambda I_1(\mathbf{e})\mathbf{g} + 2G\mathbf{e}, \tag{5}$$

$$\mathbf{e} = 1/2\left(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^{\mathrm{T}}\right), \quad \boldsymbol{v}\cdot\mathrm{T}|_{\Gamma_i} = 2\pi M_n^2 \boldsymbol{v}\big|_{\Gamma_i}.$$

Here $\mathbf{T}$, $\mathbf{e}$ and $\mathbf{g}$ are the stress, deformation and unit tensors, respectively; $\boldsymbol{u}$ and $\boldsymbol{n}$ are the point displacement and surface normal vectors, λ and G are Lamé constants; Γ_i at $i = 1, 2$ are the actual external and internal surfaces of the body. With the aid of the virtual work principle we pass to a simplified (weak) formulation of the elasticity equations.

The magnetostatic problem is solved with respect to the non-perturbed configuration of the particle in the form of a superposition of spherical functions as

$$\psi^{(1)} = (A_i r + B_i/r^2)\cos\vartheta, \quad i = 1, 2, 3,$$

where the coefficients are found from the boundary conditions and the conditions at infinity. This yields

$$\psi^{(1)} = \frac{4\pi b\chi H_0 \cos\vartheta}{\bar{r}^2}\,\frac{(1-q^3)(3+8\pi\chi)}{32\pi^2\chi^2(1-q^3)+9(1+4\pi\chi)},$$

$$\psi^{(2)} = \frac{4\pi b\chi H_0 \cos\vartheta}{\bar{r}^2} \times \frac{8\pi\chi\bar{r}^3(1-q^3) - 3(1-\bar{r}^3)}{32\pi^2\chi^2(1-q^3)+9(1+4\pi\chi)}, \tag{6}$$

$$\psi^{(3)} = \frac{32\pi^2 b\bar{r}(1-q^3)\chi^2 H_0 \cos\vartheta}{[32\pi^2\chi^2(1-q^3)+9(1+4\pi\chi)]},$$

where $\bar{r} = r/b$ and $q = a/b$ are the dimensionless radial coordinate and the cavity radius, respectively.

The elasticity problem with allowance for a non-uniform stress distribution is solved by the finite-element method, whereas the distribution of magnetization is determined with the aid of Eqs. (6). Finally, the external elongation parameter defined as

$$\varepsilon_l^{(\mathrm{ext})} = \frac{u(r=b,\ \vartheta=0)}{b}, \tag{7}$$

see Fig. 1, is evaluated. Setting $r = a$ we introduce similarly to Eq. (7) the "internal" elongation parameter $\varepsilon_l^{(\mathrm{int})}$ at the inner side of the pole. Similarly, at the equator of the body ($r = b, a$; $\vartheta = \pi/2$) the external and internal contraction parameters ε_t are introduced. Those two pairs of ε are also combined to give the polar and equatorial dimensionless increments of the wall thicknesses, δ_l and δ_t. For example,

$$\delta_l = (1-q)^{-1}\left[\varepsilon_l^{\mathrm{ext}} - q\varepsilon_l^{\mathrm{int}}\right], \quad \delta_t = (1-q)^{-1}\left[\varepsilon_t^{\mathrm{ext}} - q\varepsilon_t^{\mathrm{int}}\right].$$

All those characteristics calculated as the functions of the reduced internal cavity radius are shown in Figs. 2 and 3. The numeric values of the material parameters used for the calculation are: $\chi = 1$, $H_0/\sqrt{G} = 0.3$. For technical reasons in numeric work the material is treated