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Theory, experiment, and simulations of a symmetric arrangement of quasi-two-dimensional magnetic fluid drops

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Abstract

A theoretical and experimental investigation of a symmetrical arrangement of N quasi-two-dimensional magnetic fluid drops in an external field is carried out. We observe that when the distance between drops is smaller than about one drop diameter, the interactions between drops have a dramatic impact on the pattern formation process. In these circumstances, the final patterns that form are quite predictable. This predictability can be understood qualitatively by finding the rotational preference of the drops early in the evolution process using an energy minimization approach. To investigate the final state patterns, we perform a series of numerical experiments that demonstrate good agreement with the experiments.

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1. Introduction

When a ferrofluid drop is placed in a Hele–Shaw cell and subjected to a perpendicular magnetic field, it undergoes a fingering instability that can lead to a complex, labyrinthine structure. The formation of these patterns has been well studied for a single-domain system but very little work has been done trying to understand multiple-domain systems. In contrast to the single-domain situation, the pattern formation process becomes very predictable. Fig. 1 shows an example of the final state patterns that can result when the interactive effects are important and when they are unimportant.

The physical system to be investigated is comprised of N equal-sized ferrofluid drops of initial radius R_0 contained in a Hele–Shaw cell consisting of two glass plates separated by a distance h. These drops are symmetrically arranged at the vertices of a regular N-sided polygon.

The magnetization M is taken to be collinear with the applied field and uniform throughout each of the domains. The magnetic (self) energy for a single domain can be written as [1]

$$E_{\mathrm{mag}} = 2\pi M^2 V - M^2 h \oint \mathrm{d}s \oint \mathrm{d}s' \, \hat{\mathbf{t}} \cdot \hat{\mathbf{t}}' \Phi(R/h), \tag{1}$$

where V is the volume of the drop and

$$\Phi(\xi) = \sinh(1/\xi) + \xi - \sqrt{1+\xi^2}. \tag{2}$$

Here, the integration takes place over the (two-dimensional) boundary of the domain, $R = |\mathbf{r} - \mathbf{r}'|$ is the

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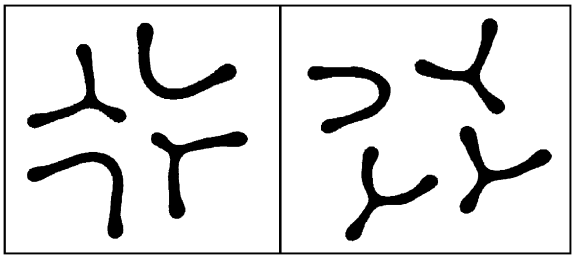

Fig. 1. Experimental final state patterns when interactions play an important role (left) and when they are insignificant (right).

distance between two points on the contour, and $\hat{\mathbf{t}}$ and $\hat{\mathbf{t}}'$ are unit tangent vectors at these points.

When considering a multi-domain system, each drop will contribute this self-energy but there will also be interaction terms of the form

$$E_{ij} = -M^2 h \oint \mathrm{d}s_i \oint \mathrm{d}s_j \hat{\mathbf{t}}_i \cdot \hat{\mathbf{t}}_j \Phi(R_{ij}/h), \tag{3}$$

where the subscripts now refer to contours on separate domains. Because we are only interested in how the interactions influence the pattern formation process, we focus our attention on the total interaction energy of the system, given by

$$E_{\mathrm{int}} = \frac{1}{2}\sum_{i\neq j} E_{ij} = \sum_{i<j} E_{ij}. \tag{4}$$

2. Energetically preferred rotational states

We are interested in finding the preferred energy states for this system early in the evolution. Thus, we introduce pure mode disturbances of the form $\varsigma_i \cos[n_i(\theta + \alpha_{ij})]$, where the mode number n_i indicates the number of bumps on domain i, and α_{ij} is the rotation angle of drop i with respect to drop j. Linearizing the interaction energy with respect to the small parameter ς_i/R_0 then gives [2]

$$E_{ij} = I_{ij}^{(0)} + \varsigma_i A_{ij} \cos n_i\alpha_{ij} + \varsigma_j A_{ji} \cos n_j\alpha_{ji} \tag{5}$$

where $I_{ij}^{(0)}$ is the interaction energy for two circular domains and A_{ij} is the *amplitude coefficient* that depends on the mode number of the ith drop (n_i) and the distance between drops i and j.

Symmetry dictates that we can, without loss of generality, focus our attention on a single domain (say, drop 1). Incorporating only that portion of the interaction energy that depends on drop 1 and writing the rotation angles as $\alpha_{ij} = \alpha_{12} + (j-2)\pi/N$, we find an expression for the extreme angles α_{12}^* that involves complicated summations involving the amplitude coefficients. However, since the interactions do not play a significant role when the drops are separated by more than about one drop diameter, it is only the two nearest neighbors that will affect any particular drop. By including only nearest-neighbor interactions and using the fact that $A_{12} = A_{1N}$, we find the following expression for the extreme angles:

$$\tan n_1\alpha_{12}^* = -\frac{\sin[(N-2)n_1\pi/N]}{1+\cos\left[(N-2)n_1\pi/N\right]}, \tag{6}$$

valid as long as the denominator is nonzero.

To guarantee we have an energetic minimum, we require the second derivative of the energy (evaluated at α_{12}^*) to be positive. This allows us to write the minimum energy rotational states for drop 1 as

$$\alpha_{12}^* = -\frac{N-2}{2N}\pi + \frac{l\pi}{n_1}, \tag{7}$$

where l is an integer that satisfies

$$2l+1 < \frac{N-2}{N} n_1 < 2l+3. \tag{8}$$

Thus, to determine the preferred orientation of our system of drops, we first calculate l using Eq. (8) and then determine the preferred angle of drop 1 from Eq. (7). The orientation of the other domains can be obtained by a similar procedure. In our case, since we assume all of the drops begin with the same initial radius, it is likely (see Ref. [3]) that all of the drops will have the same initial perturbation. Thus, for the remainder of this paper, we will assume that all drops have the same mode number n.

As an example, suppose we have a configuration with $N = 3$ and $n = 2$. We then find that $l = -1$ which yields $\alpha_{12}^* = -2\pi/3$. The preferred angles for the other two drops are then easily obtained by symmetry. Fig. 2 shows the theoretical predictions and the experimental results for three different configurations. The agreement between theory and experiment is quite good.

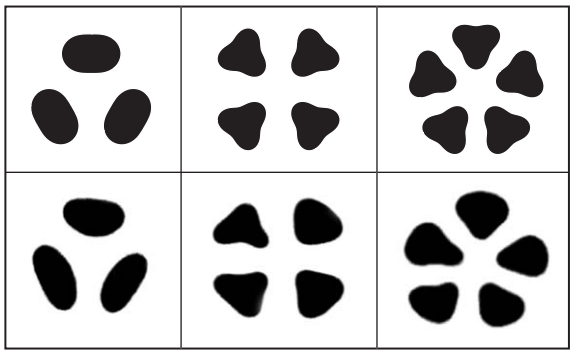

Fig. 2. Theoretical preferred states (top) and experimental realizations (bottom) for three different configurations.