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Rayleigh–Marangoni–Bénard instability of a ferrofluid layer in a vertical magnetic field

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Abstract

This paper compares the relevance of various scaling for the stability study of the Rayleigh–Bénard–Marangoni extended problem when a ferrofluid layer is submitted to a weak magnetic field normal to it. Some preliminary results are reported.

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1. Introduction

Let us consider the Rayleigh–Bénard instability for a ferrofluid submitted to an imposed magnetic field, introducing a Kelvin force coupling. It is *always* unstable with respect to the non-oscillatory instability, whether heating from above or from below [1–3]. A free deformable interface introduces explicitly the Marangoni instability [1,3]. For a Newtonian fluid layer *heated from below*, the linear Rayleigh–Bénard–Marangoni instability neglects the free surface deformation [1]. For an *isothermal* ferrofluid layer submitted to a normal field, exists a *specific static instability* [2,4], for which *the free surface will not remain flat*. Bashtovoi and his colleagues [3] and Weilepp and Brand [5] combined these two instabilities. The former considers only non-oscillatory asymptotic solutions. The later study a layer whose width is comparable to the capillary length and did not consider the Kelvin term so that the magnetic field appears only through the magnetic traction along the free interface [5]. Both teams consider *heating from below*. These are severe restrictions that do not apply to earlier results of Schwab et al. *heated from above* [6]. Since there is no prescribed velocity, reference flow values are deduced only from the prime physical parameters defining the problem. Then, one defines a proper scaling, meaningful for the phenomenon under scrutiny, which identifies also its domain of validity. Outside, the physical data do not correspond with the theoretical model. Scaling is thus a delicate operation [4].

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2. Rayleigh–Bénard inductive or non-inductive cases

An infinite horizontal layer of a ferrofluid of width d is bordered by a non-magnetic solid, at $z=0$. Its free surface Σ at $z=d$. We apply a gradient of temperature and an exterior magnetic field, both normal to the unperturbed boundaries. We look only at a *weak* magnetic field so that the magnetization is collinear with the magnetic field $\mathbf{M}=\chi\mathbf{H}$ and $\mu_0[\mathbf{H}+\mathbf{M}]=\mu\mathbf{H}$. The magnetization state equation is $M=M_0(T_0,H_0)+\chi(H-H_0)-K(T-T_0)$ near to the reference state [2,3]. The pyromagnetic coefficient $K\cong M[\alpha+1/T]$, where α is the thermal dilatation coefficient [3]. The Maxwell equations are quite easy to write [2,3], since $\mathbf{H}=\nabla\phi$. In a lot of practical cases, one neglects the temperature influence on the magnetic field so that $\nabla^2\phi=0$ defines the non-inductive case [3]. On the contrary, when the magnetic field results from a magnetic gradient, the inductive assumption gives, for low fields [2,3], $\nabla^2\phi=K'\mathbf{1}_H\cdot\nabla T$ where $K'=K/(1+\chi)$ and $\mathbf{1}_H=\mathbf{H}/H$. On both boundaries of the ferrofluid layer, the normal components of $\mu\mathbf{H}$ and the tangential component of the magnetic field $\mathbf{H}$ are continuous.

2.1. Momentum balance and Laplace–Marangoni boundary condition

Let us write in Cartesian coordinates ($i,j=1,3$), the momentum balance law for an incompressible viscous ferrofluid, submitted to an exterior magnetic field, in the gravity field [2,3]

$$\rho\,\frac{D\mathbf{v}}{Dt}=-\nabla p+\eta\nabla^2\mathbf{v}+\rho\mathbf{g}+\mu_0\mathbf{M}\nabla\mathbf{H},\tag{1}$$

where $\mathbf{v}$ is the velocity, p the generalized pressure, ρ the density, $\mathbf{g}=-g1_z$ is the gravity field (positive or negative to take into account the Rayleigh–Taylor instability [7]) and η is the kinematic viscosity. On the solid–liquid interface, all components of the velocity are equal to zero. The deformable liquid–gas interface Σ, is defined by a Monge equation linking the surface deformation to the normal component of the velocity. Along Σ, one has the Marangoni–Laplace condition [3,5,8]:

$$[T^{\mathrm{L}}_{ij}-T^{\mathrm{G}}_{ij}]_\Sigma n_j=-(\nabla_\Sigma\cdot\mathbf{n})_l\sigma(1-\delta_{il})+\delta_{il}\,\frac{\partial\sigma}{\partial x_l},\tag{2}$$

where

$$T_{ij}=-\left\{p+\frac{\mu_0}{2}\,H^2\right\}\delta_{ij}+\mu H_iH_j+\eta\left[\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right].$$

2.2. The energy balance

For most systems, the energy equation reduces to the usual form [2,3] of a Fourier equation given by $DT/Dt=\kappa\nabla^2T$. The density and the surface tension vary each linearly with temperature, so that $\rho=\rho_0[1-\alpha(T-T_0)]$ and $\sigma=\sigma_0[1-\gamma(T-T_{lg})]$ where T_{lg} is the reference liquid–gas temperature, σ_0 is the value of the surface tension at T_{lg} and $\gamma=-\sigma_0^{-1}(\mathrm{d}\sigma/\mathrm{d}T)$ is usually taken as a positive quantity. The temperature dependence will appear only in terms related to all external forces, that is body forces (given by the buoyancy term $\rho\mathbf{g}$, and the Kelvin force $\mu_0\mathbf{M}\nabla\mathbf{H}$ and the surface forces (expressed by the Laplace–Marangoni jump of stresses (2)). The solid is a perfect conductor so that along the solid–liquid border at $z=0$, $T=T_{\mathrm{wall}}=C$. Along the free deformable liquid–gas surface Σ, the heat flux is proportional to the temperature difference between the surface and the temperature T_{gas} of the gaseous phase, thus we have $-\lambda[n\cdot\nabla T]_\Sigma=a[T_\Sigma-T_{\mathrm{gas}}]$, where a is the heat transfer coefficient and λ is the thermal conductivity.

2.3. The non-inductive case [5]

Calling T_0, T_{gas}, the reference temperature at the lower solid–liquid surface and the one in the gaseous phase, the steady solution of the Fourier equation is $T=T_0-\beta z$ where $\beta=a(T_0-T_{\mathrm{gas}})/(ad+\lambda)$ can be positive or negative, depending on which interface is the heating one. We want to study the linear stability of such a reference motionless state. We use d, d^2/v, $\rho v^2/d^2$, $d\beta v/\kappa$, M_0, dM_0 to scale the length, time pressure, temperature magnetization and magnetic potential, respectively. Comparing with the classical buoyancy problem [1,3], we now take into account the Kelvin forces in Eq. (1), leading to the definition of $\Re$, the *total* Rayleigh number that expresses the bulk driving forces [3,5]:

$$\Re=\frac{\pm|g|\alpha\beta d^4}{\kappa v}+\mu_0K|\nabla H|\left[\mathbf{1}_H\cdot\frac{\nabla T}{|\nabla T|}\right]\frac{\beta d^4}{\kappa\eta}.\tag{3}$$

One could study independently buoyancy and magnetic field so that $\Re$ introduces separately the Rayleigh number [1,2] and the magnetic Rayleigh number [3], to compare their *relative* influence. In general, $\Re$ is essentially due to the constant outer magnetic gradient $|\nabla H|$. The Marangoni number is $M=-(\partial\sigma/\partial T)(\beta d^2/\eta\kappa)$. Because there is a gradient in the magnetic field, all four quadrants of the $\{\Re,M\}$ plane are now *physically* meaningful, even while keeping to a non-inductive solution.

2.4. The inductive case

The paper of Stiles et al. [9] is taking into account the inductive approximation, so that the