Available online at www.sciencedirect.com
ScienceDirect
Regional Science and Urban Economics 37 (2007) 602–617
regional SCIENCE & urban ECONOMICS
www.elsevier.com/locate/regec

Effects of quality changes in rental housing markets with indivisibilities

Tamon Ito *

Institute of Policy and Planning Sciences, University of Tsukuba, Ibaraki 305-8573, Japan

Received 21 February 2005; received in revised form 30 November 2006; accepted 15 January 2007
Available online 30 January 2007

Abstract

This paper presents comparative statics results on a rental housing market model with a finite number of apartments. The apartments are classified into categories based on their housing attributes. The apartments in a given category are treated as equals having the same rent. The main result is that when an improvement occurs in one category, the rent of the category increases, whereas rents of better categories decrease. We compare this result with another comparative statics result, and also with Braid's results in his model with a continuum of housing qualities.

JEL classification: R31; R30; D58
Keywords: Rental housing market; Comparative statics; Rent equation; Differential rent; Quality improvement

1. Introduction

In this paper, we consider comparative statics in the rental housing market model of Kaneko, Ito, and Osawa (2006), where apartments are treated as indivisible goods and are exchanged with one composite good. In particular, we consider the comparative statics of apartment rents when the quality of a certain apartment class is improved. This comparative statics will be contrasted with another result. These comparative statics give similar effects on the rent of the apartments in question but opposite effects on the rents of the other apartments.

First, we explain the background of the model as well as the relationship to the relevant literature, since the approach we take seems not to be well-known in the urban economics

* Tel./fax: +81 29 853 5580.
E-mail address: taito@shako.sk.tsukuba.ac.jp.

0166-0462/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.regsciurbeco.2007.01.002

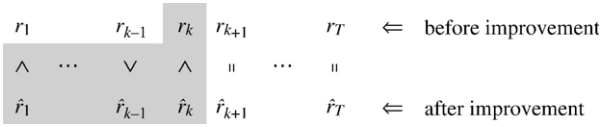

Fig. 1.

literature. Then, we give diagrammatic explanations of the two above-mentioned comparative statics results. Finally, we will mention some possible extensions of our results.

Our approach is regarded as belonging to the literature of "assignment markets" from Von Böhm-Bawerk (1921), von Neumann and Morgenstern (1944) and Shapley and Shubik (1972). In this literature, markets with one indivisible good and one divisible good have been studied, while allowing product differentiation in their indivisible units. However, the literature reflects almost no interactions with urban economics, except for Kaneko (1983) and Gerber (1985). Recently, Kaneko et al. (2006) gave a more systematic application of the approach to urban rental housing markets, and also studied various properties arising from comparative statics. The present paper is a further study in this line of research.

We also find that our approach is related to the bid rent approach in the urban economics literature originated with Alonso (1964), Muth (1969) and Mills (1972).[1] Those models are typically described by continuous variables and continuous functions. On the other hand, the main feature of our model is discreteness, i.e., each housing unit is indivisible and the number of housing units is finite. An important difference in methodology is that calculus is a main method in the standard urban economic literature, while ours is truly combinatorial.

Here, we give a brief description of our housing market model and our comparative statics. The details will be given in Sections 2–4. In our model, the apartments are classified and ranked qualitatively. Each household is assumed to have the identical utility function expressed as $h_k+g(c)$. This utility function is separable as the sum of the utility level h_k from the k-th ranked apartment itself and the utility level $g(c)$ from the c of composite good. The first term, h_k, reflects the "utility" from the housing attributes of the k-th apartment, such as size, commuting time to the central business district (CBD) and age. We interpret h_k as representing the quality of the k-th apartment.[2] Here, we can assume without loss of generality[3] that the qualities are arranged in numerical order: $h_1>h_2>\cdots>h_T$. Thus, the lower the rank, the higher the quality of the apartment.

In the rental housing market described above, we consider the competitive rents $r_1,\ldots,r_T$ of apartments 1,…, T, which are described in the first row of Fig. 1. Now, we suppose that the quality ("utility") h_k is improved to $\hat{h}_k$, but the other h_t's remain unchanged. The new competitive rents $\hat{r}_1,\ldots,\hat{r}_T$ described in the second row of the figure differ from the original competitive rents $r_1,\ldots, r_T$. In the shaded area of Fig. 1, r_k increases to $\hat{r}_k$, but the rent r_t decreases to $\hat{r}_t$ for $t<k$. It is, more or less, an assumption that the rent r_t is constant for $t>k$. This result will be given in Section 4.2 and will be proved in Section 6.

The above result will be compared with another comparative statics result, which is a variant of an earlier result in Kaneko et al. (2006). In this comparative statics, all the h_t's remain unchanged, but only the income of the household choosing the k-th apartment increases. Then, the changes in the rents $r_1,\ldots, r_T$ are expressed as shown in Fig. 2.

[1] For recent developments and treatments, see Arnott (1987) and Fujita (1989).
[2] This definition of "quality" is independent of an income. See Example in Section 2.
[3] This can be obtained by renaming the apartments under the assumption that no indifferences are allowed.