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Improved inferences for spatial regression models☆

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ABSTRACT

The quasi-maximum likelihood (QML) method is popular in the estimation and inference for spatial regression models. However, the QML estimators (QMLEs) of the spatial parameters can be quite biased and hence the standard inferences for the regression coefficients (based on *t*-ratios) can be seriously affected. This issue, however, has not been addressed. The QMLEs of the spatial parameters can be bias-corrected based on the general method of Yang (2015b, *J. of Econometrics* 186, 178–200). In this paper, we demonstrate that by simply replacing the QMLEs of the spatial parameters by their bias-corrected versions, the usual *t*-ratios for the regression coefficients can be greatly improved. We propose further corrections on the standard errors of the QMLEs of the regression coefficients, and the resulted *t*-ratios perform superbly, leading to much more reliable inferences.

1. Introduction

The maximum likelihood (ML) or quasi-ML (QML) method is popular in the estimation and inference for spatial regression models (Anselin, 1988; Anselin and Bera, 1998; Lee, 2004). However, the ML estimators (MLEs) or quasi-MLEs (QMLEs) of the spatial parameters can be quite biased (Bao and Ullah, 2007; Yang, 2015b; Liu and Yang, 2015) and hence the standard inferences for spatial effects and covariate effects, based on LM-statistics or *t*-statistics referring to the asymptotic standard normal distribution, can be seriously affected. Much effort has been devoted recently to the development of improved inference methods for the spatial econometrics models. However, most of the research has been focused on improving inferences for spatial effects in the form of point estimation (Bao and Ullah, 2007; Bao, 2013; Liu and Yang, 2015; Yang, 2015b) and testing (Baltagi and Yang, 2013a, 2013b; Robinson and Rossi, 2014a, 2014b; Yang, 2010, 2015a, 2015b). Little or no attention has been paid to the development of improved inferences for the covariate effects in the spatial regression models.

Yang (2015a) proposed a general method for constructing 2nd-order accurate bootstrap LM tests for spatial effects, but the issue of improved inferences for covariate effects was not studied. Yang (2015b) proposed a general method for 3rd-order bias and variance corrections on nonlinear estimators which are prone to finite sample bias, and argued that once the biases of nonlinear estimators are corrected, the biases of covariate effects and error standard deviations become negligible. He demonstrated the effectiveness of the methods using the linear regression model with spatial lag dependence with results showing that a 2nd-order bias correction is largely sufficient. He further demonstrated that the 2nd-order or 3rd-order corrected *t*-statistics for spatial effect indeed improve upon the standard *t*-statistics greatly, but again, no study was carried out in order to test the performance of the *t*-statistics for covariate effects, and its improvements.

Evidently, in practical applications of spatial econometrics models, it is central to have a set of reliable inference methods for the covariate effects. In this paper, we adopt the bias-correction method of Yang (2015b) to propose methods that ‘correct’ the standard *t*-statistics for

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the regression coefficients. We demonstrate that by simply replacing the QMLEs of the spatial parameters by their bias-corrected versions, the usual t-ratios for the regression coefficients can be greatly improved. We propose further corrections on the standard errors of the 'bias-corrected' QMLEs of the regression coefficients, and the resulted t-ratios perform superbly, leading to much more reliable inferences. The proposed methods are simple and can be easily adopted by practitioners. We consider in detail three popular spatial regression models: the linear regression model with spatial error dependence (SED), that with a spatial lag dependence (SLD), and that with both SLD and SED, also referred to as the SARAR model in the literature. See Anselin and Bera (1998) and Anselin (2001) for excellent reviews on these models. Bias-correction on a single spatial estimator has been considered in detail in Yang (2015b) for the SLD model, and in Liu and Yang (2015b) for the SED model. Bias-corrections for the SARAR model have not been formally considered, although briefly discussed in Yang (2015b) under a general outline for bias corrections for a model with a vector of non-linear parameters.

The line-up for the paper is as follows. Section 2 outlines the general method of bias correction on nonlinear estimators, and the methods for constructing improved t-statistics for the linear parameters in the model. Sections 3–5 study in detail the improved inference methods for the regression coefficients for, respectively, the SED model, the SLD model, and the SARAR model. Each of Sections 3–5 is accompanied with a set of Monte Carlo simulation results. Section 6 concludes the paper, and discuss further extensions of the proposed methodology.

2. Method of bias correction for nonlinear estimation

From the discussions in the introduction, it is clear that the key for an improved inference for the regression coefficients is to bias-correct the QMLEs of the spatial parameters in a spatial regression model. We now outline the method of bias correction on nonlinear estimators, not necessarily the QMLEs of the spatial parameters. In studying the finite sample properties of a parameter estimator, say $\hat{\theta}_n$, defined as $\hat{\theta}_n = \arg\{\psi_n(\theta) = 0\}$ for a *joint estimating function* (JEF) $\psi_n(\theta)$, based on a sample of size n, Rilstone et al. (1996) developed a stochastic expansion from which a bias-correction on $\hat{\theta}_n$ can be made. The vector of parameters θ may contain a set of *linear and scale parameters*, say α, and a few *non-linear parameters*, say δ, in the sense that given δ, the constrained estimator $\tilde{\alpha}_n(\delta)$ of the vector α possesses an explicit expression but the estimation of δ has to be done through numerical optimization. In this case, Yang (2015b) argued that it is more effective to work with the *concentrated estimating function* (CEF): $\tilde{\psi}_n(\delta) = \psi_n(\tilde{\alpha}_n(\delta), \delta)$, and to perform a stochastic expansion based on this CEF and hence bias corrections on the non-linear estimators defined by,

$$\hat{\delta}_n = \arg\left\{\tilde{\psi}_n(\delta) = 0\right\}, \tag{1}$$

which not only reduces the dimensionality of the bias-correction problem (a multi-dimensional problem is reduced to a single-dimensional problem if δ is a scalar parameter), but also takes into account the additional variability from the estimation of the 'nuisance' parameters α.

Let $H_{rn}(\delta) = \nabla^r \tilde{\psi}_n(\delta), r = 1, 2, 3$, be the partial derivatives of $\tilde{\psi}_n(\delta)$, carried out sequentially and elementwise with respect to δ', $\tilde{\psi}_n \equiv \tilde{\psi}_n(\delta_0)$, $H_{rn} \equiv H_{rn}(\delta_0)$, $H^\circ_{rn} = H_{rn} - \mathrm{E}(H_{rn})$, $r = 1, 2, 3$, and $\Omega_n = -[\mathrm{E}(H_{1n})]^{-1}$. Yang (2015b) presents a set of sufficient conditions under which $\hat{\delta}_n$ possesses the following third-order stochastic expansion at δ_0, the true value of δ:

$$\hat{\delta}_n - \delta_0 = a_{-1/2} + a_{-1} + a_{-3/2} + O_p(n^{-2}), \tag{2}$$

where, $a_{-s/2}$ represents terms of order $O_p(n^{-s/2})$ for $s = 1,2,3$, having the expressions,

$$\begin{aligned} a_{-1/2} &= \Omega_n \tilde{\psi}_n, \\ a_{-1} &= \Omega_n H^\circ_{1n} a_{-1/2} + \tfrac{1}{2}\Omega_n \mathrm{E}(H_{2n})(a_{-1/2} \otimes a_{-1/2}), \\ a_{-3/2} &= \Omega_n H^\circ_{1n} a_{-1} + \frac{1}{2}\Omega_n H^\circ_{2n}(a_{-1/2} \otimes a_{-1/2}) \\ &\quad + \frac{1}{2}\Omega_n \mathrm{E}(H_{2n})(a_{-1/2} \otimes a_{-1} + a_{-1} \otimes a_{-1/2}) \\ &\quad + \frac{1}{6}\Omega_n \mathrm{E}(H_{3n})(a_{-1/2} \otimes a_{-1/2} \otimes a_{-1/2}), \end{aligned}$$

with ⊗ denoting the Kronecker product.

When δ is a scalar, $a_{-s/2}$ simplifies to: $a_{-1/2} = \Omega_n \tilde{\psi}_n$, $a_{-1} = \Omega_n H^\circ_{1n} a_{-1/2} + \frac{1}{2}\Omega_n \mathrm{E}(H_{2n})(a^2_{-1/2})$, and $a_{-3/2} = \Omega_n H^\circ_{1n} a_{-1} + \frac{1}{2}\Omega_n H^\circ_{2n}(a^2_{-1/2}) + \Omega_n \mathrm{E}(H_{2n})(a_{-1/2}a_{-1}) + \frac{1}{6}\Omega_n \mathrm{E}(H_{3n})(a^3_{-1/2})$.

The key difference between the CEF-based and JEF-based expansions is that $\mathrm{E}[\tilde{\psi}_n(\delta_0)] \neq 0$ in general, but $\mathrm{E}[\psi_n(\theta_0)] = 0$, which allows a CEF-based bias correction to be derived under a more relaxed condition. Thus, a third-order expansion for the bias of $\hat{\delta}_n$ takes the form:

$$\mathrm{Bias}\left(\hat{\delta}_n\right) = b_{-1} + b_{-3/2} + O(n^{-2}), \tag{3}$$

where $b_{-1} = \mathrm{E}(a_{-1/2} + a_{-1})$ and $b_{-3/2} = \mathrm{E}(a_{-3/2})$, being respectively the second- and third-order biases of $\hat{\delta}_n$. If an estimator $\hat{b}_{-1}$ of b_{-1} is available such that $\mathrm{Bias}(\hat{b}_{-1}) = O(n^{-3/2})$, then a second-order bias-corrected estimator of δ is,

$$\hat{\delta}^{\mathrm{bc2}}_n = \hat{\delta}_n - \hat{b}_{-1}. \tag{4}$$

If estimators $\hat{b}_{-1}$ and $\hat{b}_{-3/2}$ of both b_{-1} and $b_{-3/2}$ are available such that $\mathrm{Bias}(\hat{b}_{-1}) = O(n^{-2})$ and $\mathrm{Bias}(\hat{b}_{-3/2}) = O(n^{-2})$, we have a third-order bias-corrected estimator of δ as,

$$\hat{\delta}^{\mathrm{bc3}}_n = \hat{\delta}_n - \hat{b}_{-1} - \hat{b}_{-3/2}. \tag{5}$$

An obvious approach for finding the feasible corrections $\hat{b}_{-1}$ and $\hat{b}_{-3/2}$ is to first find the analytical expressions for b_{-1} and $b_{-3/2}$ and then plugging in $\hat{\theta}_n$ for θ_0. This approach is generally not feasible for two reasons: first, it is often difficult to find these analytical expressions even for known error distributions, and second, even if these expressions are available, it may involve higher-order moments of the errors if they are nonnormal, for which estimation may be unstable numerically. To overcome this difficulty, Yang (2015b) proposed a simple and yet very effective bootstrap method to estimate the relevant expected values.

Suppose that the model under consideration takes the form

$$g(Z_n, \theta_0) = e_n,$$

and that the key quantities $\tilde{\psi}_n$ and H_{rn} can be expressed as $\tilde{\psi}_n \equiv \tilde{\psi}_n(e_n, \theta_0)$ and $H_{rn} \equiv H_{rn}(e_n, \theta_0)$, $r = 1, 2, 3$. Let $\hat{e}_n = g(Z_n, \hat{\theta}_n)$ be the vector of estimated residuals based on the original data, and $\hat{\mathcal{F}}_n$ be the empirical distribution function (EDF) of $\hat{e}_n$ (centered). When δ is a scalar parameter, the bootstrap estimates of the quantities in the bias terms are:

$$\hat{\mathrm{E}}\left(\tilde{\psi}^i_n H^j_{rn}\right) = \mathrm{E}^*\left[\tilde{\psi}^i_n\left(\hat{e}^*_n, \hat{\theta}_n\right) H^j_{rn}\left(\hat{e}^*_n, \hat{\theta}_n\right)\right], \quad i, j = 0, 1, 2, \ldots, \quad r = 1, 2, 3, \tag{6}$$

where E^* denotes the expectation with respect to $\hat{\mathcal{F}}_n$, and $\hat{e}^*_n$ is a vector of n random draws from $\hat{\mathcal{F}}_n$. To make Eq. (6) practically feasible, the following procedure can be followed.