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Shell structure in the density profile of a rotating gas of spin-polarized fermions

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Abstract

We present analytical expressions and numerical illustrations for the ground-state density distribution of an ideal gas of spin-polarized fermions moving in two dimensions and driven to rotate in a harmonic well of circular or elliptical shape. We show that with suitable choices of the strength of the Lorentz force for charged fermions, or of the rotational frequency for neutral fermions, the density of states can be tuned as a function of the angular momentum so as to display a prominent shell structure in the spatial density profile of the gas. We also show how this feature of the density profile is revealed in the static structure factor determining the elastic light-scattering spectrum of the gas.

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1. Introduction

Ultracold Fermi gases of alkali atoms such as ^{40}K or ^{6}Li in harmonic traps are quantum systems that are experimentally accessible by the techniques of atom trapping and cooling [1,2]. The high purity and the low temperature of the samples and the high resolution of the detection techniques make these systems ideal candidates for the study on the mesoscopic scale of single-level quantum properties such as shell structures in the particle density profiles [3–7]. In the experiments the trapped atomic gas can be fully spin-polarized and the strength and the anisotropy of the trap can be tuned to reach quasi-one-dimensional (1D) or quasi-two-dimensional (2D) configurations. Quantum effects in the equilibrium profiles can be greatly enhanced by varying the anisotropy of the confinement [6]. Here we show how to enhance

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quantum effects on 2D equilibrium density profiles by putting the gas into rotation.

In a wholly different physical realm electrons in a quantum dot realize a Fermi gas moving in a plane under the effect of a harmonic potential. A quantum dot is often described as an artificial atom, in which a confinement potential replaces the attractive potential of the nucleus, and the increased length scale allows access to experiments that are not practicable on real atoms [8–10]. Spin effects are also accessible and complete spin polarization is easily achieved in a small dot. Confinement by a circularly symmetric harmonic potential is of special interest, since in this case the energy needed to add electrons to the dot reveals a shell structure [11]. The effects of a magnetic field on the electronic properties of circular quantum dots have been studied by numerical diagonalization and by spin-density functional methods (see Ref. [12] and references therein). These approaches have also been applied to rectangular dot structures, when the effective lateral confinement is described as elliptical. In such a confinement the single-particle energy levels as well as the energy spectra for two-electron systems are exactly known as functions of the magnetic field [13,14].

A Fermi gas of spin-polarized charged particles in a uniform magnetic field, under conditions such that the Coulomb interactions can be neglected as specified below, can be mapped into a rotating Fermi gas of neutral atomic particles in a state of complete spin polarization, where the atom–atom interactions are negligible on account of the Pauli principle suppressing *s*-wave scattering. In this work we examine the particle density profiles of the ideal system that we have just introduced. For definiteness we shall refer throughout the paper to the ideal 2D Fermi gas of charged particles in a uniform magnetic field, although some parts of the discussion (and in particular the analysis of the elastic light-scattering spectrum given in Section 4) are specifically aimed at an atomic Fermi gas in uniform rotation.

An ideal 2D Fermi gas of charged particles in a uniform magnetic field has recently been studied by van Zyl and Hutchinson [15], who have used an inverse Laplace transform method and an expansion in Laguerre polynomials to calculate its thermodynamic properties. Their method can be implemented exactly in the case of a uniform gas, whereas recourse to a local-density approximation is needed for an inhomogeneous gas on account of the broken symmetry induced by the concurrent effects of the harmonic confinement and of the magnetic field. However, the single-particle Hamiltonian of the ideal gas can be diagonalized on the basis of left and right circular or elliptical quanta, bringing it to the form of the Hamiltonian of two independent harmonic oscillators. The eigenfunctions can thus be written in terms of Hermite polynomials, as we do in this work. Practical applications of our approach are limited to ultracold Fermi gases consisting of a restricted number of particles, so that the number of Hermite polynomials that need explicit numerical calculation remains limited. Below we shall give numerical illustrations for systems of 10 and 106 fermions at zero temperature.

The paper is organized as follows: in Section 2, we set out the single-particle Hamiltonian of the system in a generic gauge which allows its diagonalization on the basis of left- and right-handed circular or elliptical quanta. In Section 3, we express the spatial density profile of the gas in terms of Hermite polynomials and display several configurations for various values of the strength of the magnetic field and of the anisotropy parameters of the confinement. We show that an impressive shell structure arises in the profile whenever there is an excess of density of states with zero angular momentum. In Section 4, we calculate the elastic contribution to the static structure factor of the gas, in order to show how the shell structure is reflected in the elastic light-scattering spectrum. Finally, Section 5 offers some concluding remarks.

2. The model

We consider a 2D spin-polarized Fermi gas at zero temperature, made up of N non-interacting fermions of charge e and mass m_{f} subject to a uniform magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ perpendicular to the plane of confinement (the $\{x, y\}$ plane, say). The gas is immersed in a uniformly charged