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# When worlds collide: Different comparative static predictions of continuous and discrete agent models with land ☆

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ABSTRACT

This paper presents a difference in the comparative statics of general equilibrium models with land when there are finitely many agents, and when there is a continuum of agents. Restricting attention to quasi-linear and Cobb–Douglas utility, it is shown that with finitely many agents, an increase in the (marginal) commuting cost increases land rent per unit (that is, land rent averaged over the consumer's equilibrium parcel) paid by the consumer located at each fixed distance from the central business district. In contrast, with a continuum of agents, average land rent goes up for consumers at each fixed distance close to the central business district, is constant at some intermediate distance, and decreases for locations farther away. Therefore, there is a qualitative difference between the two types of models, and this difference is potentially testable.



## 1. Introduction

Models with a continuum of consumers are often employed for reasons of mathematical convenience or simplicity. Moreover, they can make precise the notion of perfect competition. As the number of agents in the world is finite, models with an infinite number of agents are not realistic unless they are close to models with a finite number of agents, in terms of equilibria and comparative statics. The scattered literature on general equilibrium models with land has tried to investigate the similarity or dissimilarity between the equilibria of these two types of models. This line of inquiry has met with limited success only; see McLean and Muench (1981), Berliant (1985, 1991), Asami et al. (1991), Kamecke (1993), Papageorgiou and Pines (1990), and Berliant and ten Raa (1991). The intuition for the dissimilarity between the models is that any partition of a σ-finite measure space, such as a Euclidean space, can have only countably many elements of positive measure. So except for a negligible set of consumers out of a continuum, all must consume or even be endowed with a set of measure zero. A corollary is that economies with a finite number of consumers approximating these continuum economies must have land consumption or endowments tending to zero almost surely.

What is the economic significance of this issue? To address this question, it is important to distinguish between models and the real world. Regarding models, the ones with land and a continuum of consumers have an inconsistency embedded in them; see Berliant (1985). Regarding the real world, when we test a model's implications with real data, we are necessarily testing implications for its analog with a finite number of consumers. Thus, if a model with a continuum of agents differs from its analog with a finite number of agents, either in terms of equilibria or comparative statics, then empirical analysis of a model with a continuum of agents is necessarily suspect, because data for a world with a continuum of agents is unavailable.

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This paper presents a dissimilarity in the comparative statics of these two types of models in the cases of quasi-linear and Cobb–Douglas utility functions. The comparative static of interest here is the effect of a change in marginal commuting cost on the per unit land rent paid, averaged over a consumer's equilibrium parcel, for the consumer who owns land at a given distance from the Central Business District (CBD). The model considered here is the standard closed city model (with an exogenous CBD and an endogenous city boundary). It is shown that when the number of agents is finite, an increase in the (marginal) commuting cost increases average land rent paid by the consumer owning land at any given distance from the CBD. In contrast, the canonical result when there is a continuum of agents is that average land rent goes up for consumers who own land at locations close to the CBD, is constant at some intermediate distance, and decreases for consumers farther away; see, for example, Fujita (1989, p. 81, Proposition 3.14, part (iii)).

This is important for both urban economic theory and empirical work. On the theoretical front, this result shows that models with a finite number of consumers are qualitatively different from models with a continuum of consumers, and therefore, in general, it is impossible to conclude that their equilibria are similar. On the empirical front, this result provides a potentially testable prediction. We shall discuss the empirical implications in the conclusions below.

Recent literature on city formation, for example Lucas and Rossi-Hansberg (2002), or the new economic geography, for example Fujita and Thisse (2002), generally employ a continuum of consumers and ordinarily have land as a commodity at least implicitly. We have postulated in our work an exogenously given CBD. In most models of city formation, the CBD or location of firms is endogenous, and there is an agglomeration externality used to determine these locations. However, these models all have embedded in them a model of consumer location and commuting, making our analysis relevant. For example, conditional on the spatial distribution of firms, one might want to consider the consumer location problem.

It is important to be precise about the particular comparative static we consider. This comparative static is exactly the same as the standard one considered by Fujita for the continuous agent model. Specifically, in either model consider a given distance from the CBD such that land at that distance is not used for agriculture. The person or persons residing at that distance from the CBD in equilibrium pays a certain average price for land (where the average is taken over their entire parcel). Now change the marginal commuting cost. If land at that fixed distance from the CBD is not used for agriculture, examine the average price for land paid by the consumer who lives there in equilibrium. This consumer might not be the same as the person who lived there in the equilibrium under the old commuting cost. How does the average price of land change? *We claim that this is the comparative static of empirical relevance, since land price as a function of distance from the CBD is generally what is observed.* Moreover, since we do not observe land prices interior to the parcel, but rather the price function averaged over the entire parcel, we study the latter. In other words, the genesis of this project was to construct the discrete model analog of Fujita's (1989) comparative static, rather than begin with our comparative static and construct the analog in the continuous agent model.

As pointed out by several readers, there is another, closely related comparative static that has the same sign for both the continuous and discrete agent models. Consider the following exercise. In either the continuous or discrete model, fix a commuting cost and equilibrium. Number the consumers from the CBD outward;[1] each pays a certain average price for their land parcel. Now change the marginal commuting cost. There is a new equilibrium. Since all consumers are identical, they can be numbered arbitrarily from the CBD outward. Maintain the order from the old equilibrium. In general, for the linear city model, the change in the average price paid in equilibrium by each consumer in the continuum and the finite models has the same sign as the change in the marginal commuting cost. We have presented this alternative comparative static for clarity, to emphasize what we are doing in this paper and what we are not doing. It is our view that this second comparative static is not of empirical relevance, since it is difficult to keep track of agents' locations after a change in commuting cost. In contrast, it is easy to keep track of the per unit price of a parcel at a given location, though the inhabitants may change with a change in commuting cost. It is clearly the case that there are comparative statics that will agree for the two models, in sign if not in magnitude, and others that differ.

The intuition behind our result is as follows. Fix an arbitrary inhabited location in the city, but close to the city edge. For the model with a continuum of agents, when commuting cost rises, in equilibrium the density of agents at the given location goes down, as they relocate to locations closer to the CBD. Thus, demand for the fixed supply of land at that location is reduced, so price goes down. In the model with a finite number of consumers, in equilibrium the number of consumers demanding land at that location is always 1. There is no margin on which to adjust density. The consumer who owns land at that location is squeezed by the increased commuting cost. If the income effect on consumption good is small, expenditure on land must go down, or its average price goes up whereas parcel size shrinks.

As a referee has pointed out, we demonstrate a difference in a comparative static between the model with a continuum of consumers and the model with a finite number of consumers. As enumerated above, many have tried various different methods to connect the two models. Some attempts involve trying to show directly that models with a continuum of agents and models with a fixed, finite number of agents have the same equilibria. Our results address these attempts. The more classical literature tries to connect the two types of models by approximating the model with a continuum of agents by taking the limit of models with a finite number of agents, where the number of agents tends to infinity. Our results also apply here, since they are independent of the number of agents in the finite model.

In the next section we introduce the notation and present the comparative static in the case of quasi-linear and Cobb–Douglas utility for the model with a finite number of consumers. This is essentially the model of Berliant and Fujita (1992) but with an endogenous city boundary that is determined using an exogenous agricultural land rent. The last section presents our conclusions. An appendix contains a complementary theorem on existence of equilibrium for the closed city model where the extent of the city is endogenous and determined by agricultural land rent.[2]

[1] There are some technical issues involved in such a numbering for the continuous model, but they will be ignored here.

[2] Although equilibrium in the quasi-linear utility case is found explicitly, the Cobb–Douglas case yields a model with no known result on existence of equilibrium. Of course, without such a result the comparative static could be vacuous.