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On the theory of the electric field and current density in a superconductor carrying transport current

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Abstract

A theory is given to explain the physics behind the flow of low-frequency ac transport current around a closed superconducting circuit, where the circuit consists of two long, straight, parallel, uniform conductors, connected to each other at one end and to an applied emf at the other end. Thus one conductor is the return path for the other. A question of interest is what drives the current at any given point in the circuit. The answer given here is a surface charge, where the purpose of the surface charge is to spread the local emf around the circuit, so that at each point in the conductor it produces, together with the electric field of the vector potential, the electric field necessary for the current to flow. But it is then necessary to explain how the surface charge gets there, which is the central problem of the present analysis. The conclusion is that the total current density consists of the superposition of a large transport current and a very much smaller current system of a different symmetry. The transport current density is defined as a two-dimensional current density with no divergence. It flows uniformly along the conductor length, but can vary over the cross-section. The small additional current density has a much different symmetry, being three-dimensional and diverging at the surface of the conductor. Based on a slightly modified Bean model the transport current is treated as supercurrent having the value $\pm J_c$, while the small additional system of current is like normal current, with a density given by the electric field divided by a resistivity. The electric field is computed from the sum of the negative time derivative of the vector potential and the negative gradient of the scalar potential due to the surface charge. It has components parallel and perpendicular to the long axis of the conductor. Thus the small normal current density has a perpendicular component which flows into or out of the surface thereby creating the surface charge. Since the circuit has charge neutrality one can picture the small system of normal current density at a given point along the conductor length, during a given half-cycle, as flowing in a perpendicular direction out of the surface and then turning to flow around the circuit to a similar point in the return path, where it turns and flows into the surface. The perpendicular component varies in strength along the length of the conductor, producing a surface charge density which varies approximately linearly along the length. Such a surface charge produces a uniform electric field inside the conductor which adds to the electric field of the vector

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potential and determines the flow of transport current. It is shown that one can arrive at the above results by a careful solution of the Maxwell equations, where the solution also satisfies the demands of the Bean model.

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1. Introduction

Maxwell's equations are assumed to apply to superconductors as well as to normal materials [1]. The problem investigated here, by means of a careful solution of Maxwell's equations, is the physics of low-frequency transport current in a closed superconducting circuit, where the current is established by a local ac applied emf. To solve these equations for the Maxwell current density one requires a material constitutive relation describing the dependence of the current density on the electric field. The Bean model, slightly modified, is used here. The circuit consists of two long straight "coated conductor" strips where the strips are connected together at one end and connected to the emf at the other end, with their length assumed to be long enough for the end connections to be ignored. The superconducting strips are parallel and separated by a distance large compared with their cross-sectional dimensions. Analysis is made for one strip, with the other taken as its return path. The Maxwell equations are more accurate than the Bean model, and therefore the approach used here is to first solve these equations in terms of the current and charge density, in the ac quasi-static approximation, before the Bean model is introduced. As used here the "quasi-static" (basically low-frequency) approximation for ac problems is defined by the range of frequencies where the d'Alembertian operator can be approximated by the Laplacian. The approximation is regarded as an approximation which eliminates retardation and radiation from the solution of Maxwell's equations, but not, as often assumed, an approximation to be introduced into the Maxwell equations themselves, i.e. the equation $\mathrm{curl}\,\mathbf{H} = \mathbf{J} + \dot{\mathbf{D}}$ is not changed to $\mathrm{curl}\,\mathbf{H} \approx \mathbf{J}$ by the quasi-static approximation. It is this approach which leads to the novel results.

The solution of Maxwell's equations is given in terms of a vector and a scalar potential, where in the Lorentz gauge the vector potential comes from the current in the circuit and the scalar potential comes from any charge density on the conductor surface, since volume charge density cannot exist at low frequencies in a uniform material. But a priori it is not obvious that any surface charge exists, and in fact for some problems it does not exist. But in a recent series of papers [2,3] it was pointed out that a surface charge is a necessity for the case of transport current established by a local emf in a closed circuit. The surface charge is responsible for "spreading" the local emf evenly around the circuit, and it explains how a local emf can cause transport current to flow. The surface charge is also responsible for meeting the field-free region demand of the Bean model. But it is then necessary to explain where the surface charge comes from and this is the focus of the present analysis. One can imagine in an initial approximation that current flowing in the circuit is a superconducting transport current that flows around the circuit, uniformly along the conductor length although not uniform over the area, and in general this current density is without divergence. The direction of the transport current density over the area of the conductor depends on the history of the electric field. If the electric field is now considered it not only determines the direction of the transport supercurrent, it also introduces an additional normal current, which is the part of particular interest here. Measurements of the voltage/current relationship are sufficient to define a critical current density which roughly estimates the changeover from superconductor to normal conductor behavior, but such measurements do not