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Chaotic dynamics of underdamped Josephson junctions in a ratchet potential driven by a quasiperiodic external modulation

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Abstract

In this article the classical dynamics of the underdamped Josephson junction in a periodic asymmetric potential of the ratchet type are considered. Driving the junction by an external quasiperiodic dual-frequency field, and in dependence on parameter values we show that the system exhibits chaotic trajectories and attractors. We also identify four voltage reversals occurring, associated with the junction bifurcation to periodic behaviour. We argue that the junction favours a directed transport accompanied by the destruction of the supercurrent state at zero voltage. The role of the incommensurability of the field frequencies and asymmetry of the potential has been discussed.

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Keywords: Josephson devices; Ratchet effect; Non-linear dynamics and fluctuation phenomena

1. Introduction

Over the last decade, the dynamical properties of non-linear systems have been extensively investigated with enormous emphasis on the transport phenomena, which formed the essential issue of the problem. The studied effect in these systems is called ratchet for which extracted usable work in the form of a directed current is feasible under non-equilibrium thermal fluctuations with unbiased conditions [1,2]. A burst of work in this regard has been triggered, motivated by the quest for understanding and explaining the mechanism of unidirectional transport in numerous ratchet systems such as biological cells, semiconductor superlattices, Josephson junctions and asymmetric

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superconducting SQUID rings [3–5]. SQUID rings are highly non-linear systems, and it has been demonstrated recently that they can be of significant importance in applications as logic and memory devices [6], when the ring responds strongly non-adiabatically to time dependent changes in its potential, taking the phenomenon of stochastic resonance into scope [7]. In addition, the Josephson junction itself which is essentially of non-linear nature [8], constitutes the key element of the superconducting ring, and has been under investigation on its own, bearing in mind that the Josephson effect enables the generation of time-varying fields under externally applied dc fields, and vice versa [9]. Several models have been proposed in order to explain the mechanism of the directed motion of a particle and the role of noise therein (see for example Refs. [1,10]). In this approach, one can view the Brownian motion of a particle in the presence of solely white noise. This leads as a result, to a diffusion of this particle that is moving in one-dimensional periodic potential under applied forces. Thus there is not in average a preferred direction to the transport of the particle. However, the inclusion of a coloured noise playing the role of input energy helps constructively the particle overcome thermal energy barriers, and in effect acquiring a directional motion. Studying these phenomena in Josephson junctions is potentially important [8], since the junction dynamical behaviour can be traced via the equation of motion of the junction, which is similar to that describing a particle in a periodic potential. Falo et al. demonstrated that trapped fluxons in overdamped Josephson junction arrays have a periodic ratchet pinning potential and behave in a well-defined directional motion [11]. Furthermore, the dynamics of the overdamped Josephson junction under the influence of an external quasiperiodic two-frequency driving have been investigated in [12]. In this case, the junction equation of motion in the absence of noise, is deterministic and the junction resembles a particle in one-dimensional spatially periodic non-ratchet (sinusoidal) potential, for which a directed particle current owing to non-vanishing mean velocity relates to a non-zero mean voltage developing across the junction (resistive state) due to symmetry breakings.

In this article, we base and elaborate our study using the asymmetric potential described by Mateos in his analysis of current reversal phenomena in deterministically rocked (i.e., in the presence of time-periodic oscillating fields) ratchets [13]. The underdamped dynamics of Josephson junctions particularly in rocking ratchets, have been rarely investigated, when an asymmetric potential and a quasiperiodic time-dependent field having incommensurate frequencies are both taken into consideration in the junction equation of motion. This is achieved by presenting data showing the dynamical orbits and phase portraits of the underdamped junction which markedly shows sensitive dependence on parameters values.

2. The equation of motion of the underdamped junction

Embarking on the resistively shunted junction (RSJ) model [8], the equation of motion can be obtained, for which we have here three current components; normal, supercurrent and displacement corresponding respectively to the shunt resistor, the Josephson element having a critical current I_C, and the capacitor. Thus, current conservation in the circuit enables writing the following non-linear equation:

$$\frac{1}{\omega_p^2}\frac{d^2\theta}{d\tau^2}+\frac{1}{\omega_c}\frac{d\theta}{d\tau}+\frac{dU(\theta)}{d\theta}=I(\tau) \qquad (1)$$

Use of the phase-voltage (phase slip) relation $d\theta/d\tau = (2eV/\hbar)$ has been made in deriving Eq. (1). The external current $I(\tau)$ is normalised by the critical current I_C, $\omega_p = \left(\sqrt{2eI_C/\hbar C}\right)$ and $\omega_c = (2eI_CR/\hbar)$ are the junction plasma frequency and the characteristic frequency respectively, and can be related to the damping parameter β of the junction by $\omega_c/\omega_p = \beta^{1/2}$. For the case in which β is very large, i.e. when $\omega_p \ll \omega_c$ the junction damping is relatively small and we can no longer ignore the "inertial" term $\left(\omega_p^{-2}\ddot{\theta}\right)$ in Eq. (1) which as a consequence, cannot be then oversimplified and we ought to consider the underdamped Josephson junction.