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On the rationale of spatial discrimination with quantity-setting firms

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ABSTRACT

We show in a game-theoretic model that when quantity-setting firms first choose whether to discriminate or not and then set quantities, the unique equilibrium consists in all firms selling a uniform quantity to all consumers. This sharply contrasts with the case of price-setting firms.

1. Introduction

A consistent body of literature originated by the pioneering work of Hamilton et al. (1989) assumes that firms are able to sell differentiated quantities to consumers which are located at different points in the space.[1] That is, spatial discrimination is assumed to be a feasible option for firms. Moreover, Hamilton et al. (1989) and the literature thereafter implicitly assume that when spatial discrimination is a feasible option, firms actually discriminate across consumers by selling location-specific quantities.[2] In other words, it is assumed that when discrimination is possible, discrimination occurs. However, this is far from being obvious. In fact, when a firm can sell location-specific quantities (i.e. discrimination is feasible), the firm can also sell the same quantity to every consumer: the possibility to discriminate implies the possibility not to discriminate (while the reverse is not always true). As a consequence, one should address the question whether discrimination is actually the equilibrium of a game in which firms choose whether to discriminate or not.

The purpose of this note is to investigate whether quantity-setting firms choose to sell discriminatory quantities when they are allowed (but they are not constrained) to do so. The analysis is performed within the Hotelling spatial framework.[3] We obtain that the unique sub-game perfect equilibrium consists in both firms selling the same quantity to all consumers: that is, *discrimination does not emerge in equilibrium*. As a consequence, the assumption that firms *can* sell location-specific quantities ("discrimination is feasible") may be not a sufficient rationale for firms actually selling location-specific quantities ("discrimination occurs").

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1 As Hamilton et al. (1989) argue, a situation where firms set location-specific quantities corresponds to the case of spatial price discrimination. In fact, when each firm chooses a location-specific quantity, the market-clearing condition determines the price at each location: therefore, prices are not uniform along the space, i.e. they are discriminatory. To avoid confusion, we shall call the situation where firms set location-specific quantities with the name of Cournot spatial discrimination. Instead, a situation where firms set location-specific prices shall be called Bertrand spatial discrimination.

2 A non-exhaustive list of articles which make this assumption comprehends Anderson and Neven (1991), Pal (1998), Mayer (2000), Chamorro-Rivas (2000), Matsushima (2001), Shimizu (2002), Yu and Lai (2003), Gross and Holahan (2003), Matsumura (2003), Gupta (2004), Gupta et al. (2004), Berenguer Maldonado et al. (2005), Matsumura and Schimizu (2005a,b), Gupta et al. (2006), Li (2006), Matsumura and Okamura (2006), Matsumura and Shimizu (2006), Pal and Sarkar (2006), Benassi et al. (2007), Matsumura and Schimizu (2008) and Ebina et al. (2009). In these papers, one can frequently read sentences like: "The firms deliver the product to the consumers and arbitrage is assumed to be infeasible. Thus the firms *can* discriminate across consumers" (Gupta et al., 2004, p. 763). The assumption of feasibility of spatial discrimination is (implicitly) supposed to be a sufficient condition also for the fact that firms *actually* discriminate across consumers.

3 The analysis within the Salop model yields identical results.

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More generally, this note can be seen as the Cournot version of the article by Thisse and Vives (1988). In a two-stage game where firms first decide whether to price discriminate or not and then set prices, Thisse and Vives (1988) show that the unique sub-game perfect equilibrium is characterized by price discrimination and that firms are trapped into a prisoner dilemma, because profits are lower under discrimination than under uniform pricing. We differentiate from Thisse and Vives (1988) by assuming that firms set quantities instead of prices. We show that, in a sharp contrast with Thisse and Vives (1988), the unique sub-game perfect equilibrium is characterized by both firms selling uniform quantities to all consumers (no discrimination occurs). However, as in Thisse and Vives (1988), this situation is a prisoner dilemma: the reason is that in the Cournot framework, price discrimination yields higher profits than uniform pricing, while the reverse holds in the Bertrand framework.

This note is structured as follows. In Section 2 the model is introduced. In Section 3 we study the equilibrium.

2. The model

As in Thisse and Vives (1988), there is a linear market of length 1. The left corner is denoted by 0, while the right corner is denoted by 1. Consumers are uniformly distributed, and their location is identified by $x \in [0, 1]$. There are two firms, firm A and firm B, whose location is identified respectively by a and b. Without loss of generality, we assume $0 \leq a \leq b \leq 1$. Firms set quantities. The market-clearing condition determines the price at each location. Arbitrage between consumers is excluded. Denote by $q_{A,x}$ and $q_{B,x}$ the quantity produced by firm A and firm B respectively at location x. If firm $J = A, B$ sells the same quantity to all consumers, then $q_{J,x} = q_{J,x'}$, $\forall x, x' \in [0, 1]$: in this case the subscript x is omitted for simplicity. At each location x, the (inverse) demand function is linear as in Hamilton et al. (1989) and others, and it is given by: $p_x = 1 - (q_{A,x} + q_{B,x})$. Each firm produces at constant marginal costs, which are normalized to zero. Fixed costs are nil, but the firms pay the transportation costs to ship the goods from the plant to the consumers' locations. We assume linear transportation costs as in Hamilton et al. (1989) and others. That is, to ship one unit of the product from its plant a (resp. b) to a consumer located at x, firm A (resp. B) pays a transport cost equal to: $t|a - x|$ (resp. $t|b - x|$), where t is the (strictly positive) unit transport cost. We assume that $t \leq 5/17$: this condition guarantees that there are no local monopolies. This assumption is standard in Cournot spatial price discrimination literature.[4] The timing of the game is the same as in Thisse and Vives (1988), with the addiction of a previous stage for the choice of locations as in Eber (1997) and Colombo (forthcoming). That is, at time 1 firms decide where to locate in the market, at time 2 firms decide whether to discriminate or not; at time 3 firms set the quantities. This yields four possible situations at time 3: both firms discriminate (DD), no firm discriminates (UU), only firm A discriminates (DU) and only firm B discriminates (UD). The profits earned by firm A at point x are given by: $\pi^i_{A,x} = (1 - q^i_{A,x} - q^i_{B,x} - t\left|a^i - x\right|)q^i_{A,x}$, the profits earned by firm B at point x are given by: $\pi^i_{B,x} = (1 - q^i_{A,x} - q^i_{B,x} - t\left|b^i - x\right|)q^i_{B,x}$, and the overall profits of firm A and firm B are respectively: $\Pi^i_A = \int_0^1 \pi^i_{A,x}\mathrm{d}x$ and $\Pi^i_B = \int_0^1 \pi^i_{B,x}\mathrm{d}x$, where $i = DD, UU, DU, UD$. To save notation, in the following the superscript to the locations is omitted. We solve the game by backward induction.

3. The equilibrium

As usual, we start from the last stage of the game. First, we calculate the equilibrium quantity schedules when both firms discriminate (case DD). We directly refer to Proposition 1 in Hamilton et al. (1989). Therefore:

$$q^{DD*}_{A,x} = (1 - 2t|a - x| + t|b - x|)/3 \tag{1}$$

$$q^{DD*}_{B,x} = (1 - 2t|b - x| + t|a - x|)/3. \tag{2}$$

Substituting (1) and (2) into Π^{DD}_A and Π^{DD}_B, we get:

$$\Pi^{DD*}_A = \frac{3 - 3t\left(1 - 4a + 4a^2 + 2b - 2b^2\right) + t^2\left[1 + 4a^3 + 12a^2(1 - b) + 3b + 3b^2 - 4b^3 - 6a\left(1 + 2b - 2b^2\right)\right]}{27}$$

$$\Pi^{DD*}_B = \frac{3 - 3t\left[2a - 2a^2 + (1 - 2b)^2\right] + t^2\left[1 + 4a^3 + a^2(3 - 12b) - 6b + 12b^2 - 4b^3 + 3a(1 - 2b)^2\right]}{27}.$$

Now we consider the case where no firm discriminates across consumers (case UU). The profit functions of firm A and firm B are: $\Pi^{UU}_A = \int_0^1 (1 - q^{UU}_A - q^{UU}_B - t|a - x|)q^{UU}_A \mathrm{d}x$ and $\Pi^{UU}_B = \int_0^1 (1 - q^{UU}_A - q^{UU}_B - t|b - x|)q^{UU}_B \mathrm{d}x$, which are concave in the quantity. Therefore, the equilibrium quantities are the solution of the system composed by $\partial \Pi^{UU}_A / \partial q^{UU}_A = 0$ and

4 See Hamilton et al. (1989) for a discussion of this assumption. Our assumption is more stringent than in Hamilton et al. (1989) and others, where $t \leq 1/2$. The reason is that we do not limit the analysis to the case of both firms selling discriminatory quantities: when only one firm discriminates, local monopolies are more likely to arise and a more stringent assumption on t is needed.