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On the projectile-target duality of the color glass condensate in the dipole picture ☆

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Abstract

Recently Kovner and Lublinsky proposed a set of equations which can be viewed as dual to JIMWLK evolution. We show that these dual equations have a natural dipole-like structure, as conjectured by Kovner and Lublinsky. In the high energy large N_c limit these evolution equations reduce to equations previously derived in the dipole model. We also show that the dual evolution kernel is scheme dependent, although its action on the weight functional describing a high energy state gives a unique result.

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1. Introduction

The Balitsky–JIMWLK equation [1–4] are equations governing the small-x QCD evolution for dense partonic systems. The Balitsky equations are an infinite hierarchy of coupled equations expressing the energy dependence of the scattering of high energy quarks and gluons (represented by Wilson lines in the fundamental and adjoint representations, respectively) on a target. The JIMWLK equation is a functional Fokker–Planck equation [4,5] for the small-x evolution of

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the target wavefunction equivalent to the Balitsky equations. The Kovchegov [6] equation is a simplified version of the Balitsky equations where correlations are suppressed, leading to a relatively simple non-linear equation for the elastic scattering amplitude.

It has recently [7,8] been realized that the Balitsky–JIMWLK equations miss some essential ingredients in satisfying unitarity constraints in a realistic manner. While these equations accurately handle the recombination of gluons when the gluon occupation number is large they do not properly create the growth of the occupation number starting from a dilute system. For that reason they are accurate, at least in a limited energy domain, when starting with a dense wavefunction, such as that of a big nucleus, but they are not accurate starting from a dilute system such as an elementary dipole.

Iancu and Triantafyllopoulos [7,8] suggested a new equation which consists of the Balitsky hierarchy along with a stochastic term which, in a dipole language, corresponds to dipole creation or dipole splitting. In Ref. [9] Mueller, Shoshi and Wong cast this equation into an equation for the JIMWLK weight function with the addition to the usual JIMWLK Fokker–Planck term being a fourth order functional derivative. Later on in the paper, this extension to the JIMWLK equation will be referred to as the MSW term. The effect of this stochastic term on the saturation momentum and the scattering matrix at asymptotic rapidities has been worked out in [10,11]. Finally, in Ref. [12] both the splitting and recombination terms were written in a compact and simple form in the relevant large N_c limit for the high-energy scattering problem. (Parts of these results were anticipated in Ref. [13].)

Kovner and Lublinsky [14,15] have suggested a general duality between the equations for low-density and high-density systems. In the above mentioned large N_c limit this duality is manifest [12] and the equations for dipole splitting are exactly those given in Refs. [8,9]. The general form of the duality suggested in Refs. [14,15] is likely correct and there is an ongoing effort to make this duality more explicit [16].

In this paper, we address the relationship between the splitting term discussed in Refs. [8,9] and the more general splitting terms given in Ref. [15]. This is partly an elaboration of discussions previously given in Refs. [17–19] in a closely related context. In particular, we emphasize that the splitting term used in Refs. [8,9] actually defines what is meant by the large-N_c limit in a high-energy scattering problem. We recall that for the high-energy scattering of two dipoles the leading terms in $\alpha_s N_c Y$ are in fact just the BFKL parts of the evolution. All multi-BFKL evolutions (multi-pomeron terms) are, strictly speaking, higher order in N_c. However, it is convenient to define a high-energy large-N_c limit where one keeps $1/N_c^2$ terms which are enhanced by a factor $\exp[(\alpha_P - 1)Y]$ [17–19] where Y is the rapidity of the scattering and α_P the usual hard pomeron intercept. This is a natural definition since it is exactly when $\alpha_s^2 \exp[(\alpha_P - 1)Y]$ is the order of one that unitarity corrections become important. In this large-N_c limit there is no difference between the dipole splitting used in Refs. [8,9] and the more general splitting given in Refs. [14,15].

While from the point of view of an evolved wavefunction the dominance of the "large-N_c" dipole splittings is manifest it is not so clear how this occurs as one evolves the general vertex (dipole splitting). We examine both generally and explicitly the scattering to two projectile dipoles on a target dipole. At low energies the general splitting term and the "large-N_c" splitting term are not the same and the difference between the two contributions is of the same size as either contribution [15]. However, as one evolves to large rapidity the dominant contribution, involving the "large-N_c" splitting term, behaves as $\alpha_s^4 \exp[2(\alpha_P - 1)Y]$ while the difference between the large-N_c term and the general term behaves as $\alpha_s^4 \exp[(\alpha_P - 1)Y]$. This is a remarkable result. It has a counterpart in pomeron splitting language where Braun and Vacca [20] observed that in a particular, and natural, scheme for defining the triple pomeron vertex [21,22] that vertex dom-