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Origin of chaos in the spherical nuclear shell model: Role of symmetries

T. Papenbrock [a,b,*], H.A. Weidenmüller [c]

[a] *Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996, USA*
[b] *Physics Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*
[c] *Max-Planck-Institut für Kernphysik, D-69029 Heidelberg, Germany*

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Abstract

To elucidate the mechanism by which chaos is generated in the shell model, we compare three random-matrix ensembles: the Gaussian orthogonal ensemble, French's two-body embedded ensemble, and the two-body random ensemble (TBRE) of the shell model. Of these, the last two take account of the two-body nature of the residual interaction, and only the last, of the existence of conserved quantum numbers like spin, isospin, and parity. While the number of independent random variables decreases drastically as we follow this sequence, the complexity of the (fixed) matrices which support the random variables, increases even more. In that sense we can say that in the TBRE, chaos is largely due to the existence of (an incomplete set of) symmetries.

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* Corresponding author.
E-mail address: tpapenbr@utk.edu (T. Papenbrock).

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1. Introduction and motivation

The analysis of nuclear spectra has produced ample evidence for chaotic motion. Indeed, near neutron threshold, the spectra of medium-weight and heavy nuclei display fluctuations which agree with those of random matrices drawn from the Gaussian orthogonal ensemble (GOE) [1]. Similar agreement has been found for nuclei in the sd-shell (both in experimental data [2] and in shell-model calculations [3]), and in the ground-state domain of heavier nuclei [4], although here there exists strong evidence, too, for regular motion as predicted by the shell model and the collective models. Calculations in Ce [5] have produced similar evidence for chaotic motion in atoms. Thus, chaos appears to be an ubiquitous feature of interacting many-body systems. What is the origin of this behavior? In the present paper, we address aspects of this question.

We do so using the nuclear shell model, a theory with a mean field and a residual two-body effective interaction V. (We do not include three-body forces, although there is evidence [6] that these may be needed to attain quantitative agreement with data. It will be seen that qualitatively, our arguments would not change with the inclusion of such forces.) In many nuclei, the mean field is (nearly) spherically symmetric. Thus, single-particle motion is largely regular. Chaos in nuclei seems a generic property and, hence, must be due to V. We focus attention entirely upon the effects of V. Therefore, we assume that we deal with a single major shell in which the single-particle states are completely degenerate and in which there is a fixed number of valence nucleons. (A lack of complete degeneracy would reduce the mixing of states due to V and, thus, drive the system towards regular motion.) Generic results are expected to be independent of the details of V. Therefore, we assume that the two-body matrix elements (TBME) of V are uncorrelated Gaussian-distributed random variables with zero mean value and unit variance. Our results then apply to almost all two-body interactions with the exception of a set of measure zero. (The integration measure is the volume element in the parameter space of the TBME.) The resulting random-matrix model is commonly referred to as the two-body random ensemble (TBRE) [7,8]. We ask: How does V produce chaos in the framework of the TBRE?

The two-body interaction V has two characteristic features. (i) It connects pairs of nucleons. (ii) It possesses symmetries: it conserves spin, isospin, and parity. We wish to elucidate the role of both features in producing chaos in nuclei.

The relevance of the first feature is brought out by comparing the TBRE with the GOE. We recall that in the latter, the matrix elements of the Hamiltonian couple every state in Hilbert space to every other such state. These matrix elements are assumed to be uncorrelated random variables. In the context of many-body theory, such independent couplings between all pairs of states can be realized only in terms of a many-body interaction the rank of which equals the number of valence particles. Put differently, with N the dimension of the Hamiltonian matrix, the number of independent random variables in the GOE is $N(N+1)/2$ and, for $N \to \infty$, grows much faster than N. Thus, it is intuitively clear that the GOE Hamiltonian will produce a thorough mixing of the basis states which is tantamount to chaos. In contradistinction, the number of independent two-body matrix elements in a single shell with half-integer spin j is only $j + 1/2$ while the number of many-body states with fixed total spin J grows with j like j^{m-3} where m is the number of valence particles. (The simple estimates leading to these statements are given in Appen-