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Symmetry in nuclei and beyond

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The concepts of dynamic symmetry, dynamic supersymmetry and critical symmetry are briefly reviewed. These concepts provide classification schemes for finite quantal systems. They have led to important discoveries in a variety of fields. The discovery of supersymmetry in nuclei is reviewed.

1. Introduction

In the last 30 years, the concepts of dynamic symmetry, dynamic supersymmetry and critical symmetry have played a major role in Nuclear Physics. These concepts have also been applied to other mesoscopic systems, especially molecules and finite polymers, and could in principle be applied to yet other systems both mesoscopic (atomic clusters, macromolecules, ...) and non (cuprate high-T_c materials, ...). In this article, a review of applications of symmetry methods to Nuclear Physics will be presented.

2. Symmetry

Symmetry is a wide-reaching concept used in a variety of ways. Among these, particularly important are:

(a) Geometric symmetries

These symmetries describe the geometric arrangement of constituent particles into a structure, for example atoms in a molecule. They are described by point groups, $C_n, D_n, ...$

(b) Space-time symmetries

These symmetries fix the form of the equations governing the motion of the constituent particles. For example, the form of the Dirac equation for a relativistic spin-$\frac{1}{2}$ particle

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0 \tag{1}$$

is fixed by Lorentz invariance. Space-time symmetries are described by continuous groups, here $SO(3,1)$.

(c) Gauge symmetries

These symmetries fix the form of the interaction between constituent particles and/or with external fields. For example, the form of the Dirac equation in an external electromagnetic field A_μ

$$[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi(x) = 0 \tag{2}$$

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is dictated by the gauge symmetry of the electromagnetic interaction. Since the laws of electrodynamics are invariant under the gauge trasformation $A'_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \Lambda(x)$, the gauge group is $U(1)$. It has been a major achievement of Physics to show that the fundamental (strong, weak and electromagnetic) interactions are all governed by gauge symmetries $SU_c(3) \otimes SU(2) \otimes U(1)$.

(d) Dynamic symmetries

These symmetries fix the interaction between constituent particles and/or external fields and determine the spectral properties of quantum systems (patterns of energy levels). They were introduced implicitly by Pauli in 1926 [1]. Pauli noted that the Hamiltonian for a particle in a Coulomb potential is invariant under a group of transformations larger than the rotation group, generated by the three components of the angular momentum and of the Runge-Lenz vector. In fact, it can be written in terms of the quadratic invariant (Casimir) operator of $SO(4)$

$$H = \frac{p^2}{2m} - \frac{e^2}{r} = -\frac{A}{C_2(SO(4)) + 1}. \tag{3}$$

This leads to the Bohr formula

$$E(n, \ell, m) = -\frac{A}{n^2}, \tag{4}$$

where the energy levels are written explicitly in terms of the quantum numbers characterizing the states uniquely. Since this is a symmetry of the interaction it has been called dynamic.

Dynamic symmetries assumed an important role in physics with the discovery of flavor symmetry in the 1960's by Gell'Mann [2] and Ne'eman [3]. Here the mass operator of hadrons is expanded in terms of linear and quadratic invariants of the isospin and hypercharge subgroups of the flavor group $SU_f(3)$

$$M = a + b\,[C_1(U(1)] + c\left[C_2(SU(2)) - \frac{1}{4}C_1^2(U(1))\right]. \tag{5}$$

This leads to the Gell'Mann-Okubo mass formula

$$M(Y, I, I_3) = a + bY + c[I(I+1) - \frac{1}{4}Y^2], \tag{6}$$

where again the energies of the system are given explicitly in terms of quantum numbers. An example of flavor symmetry is shown in Fig.1. The presence of the dynamic symmetry produces an ordered spectrum.

3. Dynamic symmetry

Dynamic symmetries have been used extensively in the last 25 years and have led to many important discoveries in a variety of fields. It has been found that many complex systems display dynamic symmetries. One of the best studied cases is that of atomic nuclei. Dynamic symmetries in nuclei have ben studied mostly within the framework of the Interacting Boson Model [4,5]. In this model, even-even nuclei are described as