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Nuclear Physics B 730 (2005) 164–178
NUCLEAR PHYSICS B

A dual gravity study of the $(2+1)$D compact U(1) gauge theory coupled with strongly interacting matter fields

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Received 25 May 2005; received in revised form 2 September 2005; accepted 26 September 2005

Available online 10 October 2005

Abstract

We consider the D2-brane probe action in the gravity background dual to N coincident Dp-branes by treating the separation between the D2- and Dp-branes as a non-dynamical parameter for $p = 2, 4, 6$. The gauge coupling, the core size of a non-BPS instanton and the mass gap of the compact U(1) gauge theory in the D2-brane are determined as a function of the separation in the type IIA gravity region. The results are interpreted in terms of the $(2+1)$D U(1) gauge theory coupled with the matter fields which are also strongly coupled with the $(p+1)$D SU(N) gauge field. It is shown that strong coupling of the matter fields to the SU(N) gauge field can drastically modify their screening of the U(1) gauge field. The non-perturbative dependence of the U(1) gauge coupling on the energy scale is obtained.

PACS: 11.15.Pg; 11.25.Tq; 11.10.Kk

Keywords: 3D compact U(1) gauge theory; Large-N limit; String-YM correspondence

1. Introduction

Polyakov have shown that there is no deconfinement phase for the pure $(2+1)$D compact U(1) gauge theory [1]. In the confinement phase instantons proliferate and the gauge field acquires a mass gap. After the seminal work [1] a good deal of theoretical efforts have been devoted to

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2005.09.033

the question of how the presence of matter field with fundamental charge modifies the dynamics of the U(1) gauge field [2–13]. The dynamics of the U(1) gauge field crucially depends on the number and the dynamics of the matter fields [9,10,13–16]. Theoretical analysis is most feasible if there are a large number of matter fields. One-loop calculations show that the gauge coupling is renormalized to be $g^2 \sim \frac{\Lambda}{N}$ with N, the number of matter fields and Λ, the mass of the matter fields [16]. Consequently the instanton acquires a large scaling dimension ($\sim N$) and becomes irrelevant at the critical point in the limit $\Lambda \to 0$ [9,10,13–15]. Then it is interesting to ask how a change in the dynamics of matter field affects the dynamics of the U(1) gauge field. The self-interaction of massive matter fields was shown to qualitatively modify the short distance potential between test charge in the non-compact $(2+1)$D quantum electrodynamics [17,18]. An alternative way of modifying the dynamics of matter fields is to put the matter fields under a strong additional gauge interaction. In this paper, we are going to consider a system of $(2+1)$D U(1) gauge theory coupled with matter fields in $(2+1)$D where the matter fields in turn interact strongly with a SU(N) gauge field in $(p+1)$D. (Here the $(2+1)$D space–time is a subspace of the $(p+1)$D space–time with $p = 2, 4, 6$.) When $p = 4, 6$ ($p = 2$) the SU(N) gauge coupling becomes weak at low (high) energy. In this regime the theory reduces to the aforementioned $(2+1)$D U(1) gauge theory coupled with matter fields. Then how will the dynamics of the $(2+1)$D U(1) gauge field be modified at high (low) energy for $p = 4, 6$ ($p = 2$) where the SU(N) gauge coupling becomes strong? Usual perturbative picture is not suitable to describe the strong coupling effect. The aim of the present paper is to examine the non-perturbative effect of the strong SU(N) gauge coupling on the $(2+1)$D U(1) gauge field.

For some strongly coupled gauge theories, including the one under consideration, it is advantageous to use dual string theory [19]. The exact duality between gauge and string theories has been anticipated from the observation that the Wilson loop in gauge theory satisfies a loop equation of string [20]. The first concrete example for this idea was conjectured as a duality between the type IIB string theory in the anti-de Sitter space and $\mathcal{N} = 4$ supersymmetric SU(N) gauge theory in $(3+1)$D [21–23]. The duality has opened a variety of possibilities for a new understanding on many strong coupling phenomena of gauge theories [19]. From the dual gravity description the confining nature of the $(2+1)$D SU(N) gauge theory has been confirmed [24]. Recently the idea has been applied to construct QCD-like gauge theory including fundamental matter fields [25–30]. Most recently dual gravity backgrounds have been found for an infinite family of quiver gauge theories [31].

The field theory of our interest is a non-supersymmetric theory. It contains a $(p+1)$D SU(N) gauge theory with matter fields in the adjoint representation of the SU(N) gauge group, and a U(1) gauge theory that lives on a $(2+1)$D subspace. It also contain matter fields on the $(2+1)$D subspace that carry fundamental charges for both U(1) and SU(N) gauge fields. To understand the dynamics of the U(1) gauge field, we would like to integrate out the SU(N) gauge field and the matter fields to obtain an effective theory of the U(1) gauge field. However, this is not easy to do in the strong coupling limit. In this paper, we like to show that, in the large N limit, we can obtain the effective action using a duality relation between the above field theory and D-brane in superstring theory.

The above $(2+1)$D/$(p+1)$D U(1)/SU(N) gauge theory has a dual description in terms of superstring theory where we consider a probe D2-brane lying parallel to a large number of Dp-branes in type IIA superstring theory. However, the full field theory describing the brane system is larger than the field theory of our interest. Fortunately, it is possible to study a reduced field theory from the brane configuration in the probe limit, as will be explained below. We first