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Algebraic models for the hierarchy structure of evolution equations at small x

P. Rembiesa [a], A.M. Staśto [b,c]

[a] *Physics Department, The Citadel, Charleston, SC 29409, USA*
[b] *Nuclear Theory Group, Brookhaven National Laboratory, Upton, NY 11973, USA* [1]
[c] *Institute of Nuclear Physics PAN, Radzikowskiego 152, Kraków, Poland*

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Abstract

We explore several models of QCD evolution equations simplified by considering only the rapidity dependence of dipole scattering amplitudes, while provisionally neglecting their dependence on transverse coordinates. Our main focus is on the equations that include the processes of pomeron splittings. We examine the algebraic structures of the governing equation hierarchies, as well as the asymptotic behavior of their solutions in the large-rapidity limit.

1. Introduction

Parton saturation and the unitarity bound are among the interesting, yet still not fully resolved problems of the high-energy QCD. There are two equivalent descriptions of parton saturation effects: the Balitsky hierarchy, [1] and the JIMWLK functional equation [2]. The Balitsky hierarchy consists of an infinite set of coupled evolution equations for the operators involving Wilson lines. They describe quark and gluon scattering off a target, and generally are applicable in the high-energy regime. On the other hand, the JIMWLK equation is a functional equation which describes the evolution of the target in the limit of

E-mail address: stasto@quark.phy.bnl.gov (A.M. Staśto).
[1] Permanent address.

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small x. When the JIMWLK Hamiltonian acts onto a dipole operator consisting of Wilson lines in the fundamental representation, the Balitsky hierarchy is reproduced. In the limit of negligible correlations within the target, the first equation of the Balitsky hierarchy decouples, and becomes equivalent to the non-linear Kovchegov equation [3]. The latter has been derived within the dipole formalism [4] by taking into the account multiple scatterings between the dipoles in the projectile wavefunction and the target.

A common feature of the above approaches is that they involve triple interactions of BFKL pomerons, [5] and thus describe the evolution of the system as a non-linear process. However, it has been recently recognized that the Balitsky–JIMWLK equations incorporate only pomeron mergings, but not pomeron splittings [6]. In consequence, in the course of the evolution (understood as a process in the rapidity parameter) the number of pomerons can only decrease. Therefore the above equations ignore contributions from fluctuations from any intermediate pomeron loops. However, it is well known that such terms are relevant and can be substantial, therefore they should not be neglected [7,8]. Accordingly, considerable efforts have been lately undertaken towards a better understanding of the role of the fluctuations, [9,10] by including the missing pomeron splittings in the Balitsky–JIMWLK equations [6,11–13].

Let us first consider the evolution equations as proposed in [6]. They describe the evolution of the amplitudes $T_Y^{(k)}$ for the scattering of k quark–antiquark dipoles off a target in the large number of colors $N_c \to \infty$ regime. The rapidity $Y = \ln 1/x$ substitutes for the (imaginary) time parameter of the evolution. The first two equations of the infinite hierarchy read

$$\frac{dT_Y^{(1)}(\mathbf{x},\mathbf{y})}{dY} = \bar{\alpha}_s \int d^2\mathbf{z} \big[\mathcal{M}(\mathbf{x},\mathbf{y},\mathbf{z}) \otimes T_Y^{(1)}(\mathbf{x},\mathbf{y}) - K_1(\mathbf{x},\mathbf{y},\mathbf{z}) T_Y^{(2)}(\mathbf{x},\mathbf{z};\mathbf{z},\mathbf{y})\big],$$

$$\begin{aligned}
&\frac{dT_Y^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2)}{dY} \\
&\quad = \bar{\alpha}_s \int d^2\mathbf{z} \big[\mathcal{M}(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}) \otimes T_Y^{(2)}(\mathbf{x}_1,\mathbf{y}_1;\mathbf{x}_2,\mathbf{y}_2) \\
&\qquad + K_2(\mathbf{x}_1,\mathbf{y}_1,\mathbf{x}_2,\mathbf{y}_2,\mathbf{z}) \otimes T_Y^{(1)}(\mathbf{x}_1,\mathbf{y}_1) - K_1(\mathbf{x}_1,\mathbf{y}_1,\mathbf{z}) T_Y^{(3)}(\cdots)\big].
\end{aligned} \tag{1}$$

The arguments of the dipole amplitudes $T_Y^{(k)}$ are two-dimensional vectors $\mathbf{x}$ and $\mathbf{y}$ that give the location of the dipoles in the position space. The value of the strong coupling constant $\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$ remains fixed, consistent with the leading $\ln 1/x$ approximation. The functions $\mathcal{M}$, K_1 and K_2 describe the BFKL kernel and the Pomeron vertices in the dipole picture, respectively. Their exact forms can be found in [6]. Note that the first equation is coupled through $T_Y^{(2)}$ to the second one, the second one is coupled through $T_Y^{(3)}$ to the third one, and so on. This way the infinite hierarchy is formed.

Describing the exact behavior of the solutions to this hierarchy poses a non-trivial task. Even in the case of the closed non-linear Kovchegov equation, the exact analytical solution is not available, although one can gain some partial insight about its behavior from analytic properties of various approximate cases [14–16], or numerical simulations (see, for example, [17]).