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Nuclear Physics B 707 (2005) 56–68

NUCLEAR PHYSICS B

Coupling constant dependence of the shear viscosity in $\mathcal{N}=4$ supersymmetric Yang–Mills theory

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Received 6 October 2004; accepted 24 November 2004

Available online 8 December 2004

Abstract

Gauge theory–gravity duality predicts that the shear viscosity of $\mathcal{N}=4$ supersymmetric $SU(N_c)$ Yang–Mills plasma at temperature T in the limit of large N_c and large 't Hooft coupling $g_{\rm YM}^2 N_c$ is independent of the coupling and equals to $\pi N_c^2 T^3/8$. In this paper, we compute the leading correction to the shear viscosity in inverse powers of 't Hooft coupling using the α'-corrected low-energy effective action of type IIB string theory. We also find the correction to the ratio of shear viscosity to the volume entropy density (equal to $1/4\pi$ in the limit of infinite coupling). The correction to $1/4\pi$ scales as $(g_{\rm YM}^2 N_c)^{-3/2}$ with a positive coefficient.

PACS: 04.70.Dy; 11.25.-w; 11.10.Wx

Keywords: AdS/CFT correspondence; Thermal field theory

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2004.11.055

1. Introduction

Transport coefficients such as viscosity, diffusion constants, thermal and electric conductivities are the key ingredients in describing the hydrodynamic regime of any medium [1]. These coefficients are usually obtained from experiment rather than computed from first principles of an underlying microscopic theory, because the study of realistic strongly interacting systems often remains beyond the reach of currently available theoretical methods.

For finite-temperature quantum field theories, and thermal gauge theories in particular, computations based on the Boltzmann equation in the regime of weak coupling $g \ll 1$ show [2–5] that the shear viscosity behaves as

$$\eta \sim \frac{C N_c^2 T^3}{g^4 \log 1/g^2}, \tag{1.1}$$

where N_c^2 is the number of colors and C is a large numerical coefficient.

Since the entropy scales as $S \sim N_c^2 T^3 V_3$, the ratio of shear viscosity to the volume entropy density $s = S/V_3$ behaves as

$$\frac{\eta}{s} \sim \frac{1}{g^4 \log 1/g^2} \gg 1 \tag{1.2}$$

in the regime of weak coupling. On the other hand, hydrodynamic models used to describe elliptic flows observed in recent heavy ion collision experiments at RHIC seem to favor small values of the ratio η/s [6–8]. This is not a contradiction, however, since the experimental data were obtained for the range of energies where the coupling remains relatively large, whereas the result (1.2) is valid for small coupling.

It is therefore desirable to obtain results for the viscosity–entropy ratio in the regime of intermediate and strong coupling. Lattice simulations cannot address the issue of real-time dynamics directly facing (among other things) a formidable problem of analytic continuation. (For indirect approaches, see [9–12].)

In the absence of more conventional methods, the AdS/CFT (or gauge theory/gravity) duality conjecture in string theory [13–15] emerged as a source of insights into the non-perturbative regime of thermal gauge theories. The best studied example of the duality relates a four-dimensional finite-temperature $\mathcal{N} = 4$ $SU(N_c)$ supersymmetric Yang–Mills (SYM) theory in the limit of large N_c and large 't Hooft coupling $g_{\mathrm{YM}}^2 N_c$ to the supergravity background corresponding to a stack of N_c near-extremal black three-branes.

Using the AdS/CFT conjecture, one is able to predict the behavior of the entropy of $\mathcal{N} = 4$ SYM in the regime of strong coupling [16,17]. In the large N_c limit, the entropy is given by

$$S = \frac{2\pi^2}{3} N_c^2 V_3 T^3 f\big(g_{\mathrm{YM}}^2 N_c\big), \tag{1.3}$$

where the function $f(g_{\mathrm{YM}}^2 N_c)$ interpolates (presumably smoothly) between 1 at zero coupling and $3/4$ at infinite coupling. The strong coupling expansion for f was obtained by Gubser, Klebanov and Tseytlin [17]

$$f\big(g_{\mathrm{YM}}^2 N_c\big) = \frac{3}{4} + \frac{45}{32}\zeta(3)\big(2g_{\mathrm{YM}}^2 N_c\big)^{-3/2} + \cdots. \tag{1.4}$$