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PHYSICS LETTERS B

Physics Letters B 631 (2005) 1–6

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Modified Gauss–Bonnet theory as gravitational alternative for dark energy

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Received 16 September 2005; received in revised form 4 October 2005; accepted 4 October 2005

Available online 10 October 2005

Editor: T. Yanagida

Abstract

We suggest the modified gravity where some arbitrary function of Gauss–Bonnet (GB) term is added to Einstein action as gravitational dark energy. It is shown that such theory may pass solar system tests. It is demonstrated that modified GB gravity may describe the most interesting features of late-time cosmology: the transition from deceleration to acceleration, crossing the phantom divide, current acceleration with effective (cosmological constant, quintessence or phantom) equation of state of the universe.

PACS: 98.80.-k; 98.80.Es; 97.60.Bw; 98.70.Dk

1. The explanation of the current acceleration of the universe (dark energy problem) remains to be a challenge for theoretical physics. Among the number of the approaches to dark energy, the very interesting one is related with the modifications of gravity at large distances. For instance, adding $1/R$ term [1,2] to Einstein action leads to gravitational alternative for dark energy where late-time acceleration is caused by the universe expansion. Unfortunately, such $1/R$ gravity contains the instabilities [3] of gravitationally bound objects. These instabilities may disappear with the account of higher derivative terms leading to consistent modified gravity [4]. Another proposals for modified gravity suggest $\ln R$ [5] or $\mathrm{Tr}\, 1/R$ terms [6], account of inverse powers of Riemann invariants [7] or some other modifications [8]. The one-loop quantization of general $f(R)$ in de Sitter space is also done [9]. In addition to the stability condition which significantly restricts the possible form of $f(R)$ gravity, another

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doi:10.1016/j.physletb.2005.10.010

restriction comes from the study of its Newtonian limit [10]. Passing these two solar system tests leads to necessity of fine-tuning of the form and coefficients in $f(R)$ action, like in consistent modified gravity [4]. That is why it has been even suggested to consider such alternative gravities in Palatini formulation (for recent discussion and list of references, see [11]).

In the present Letter we suggest new class of modified gravity, where Einstein action is modified by the function $f(G)$, G being Gauss–Bonnet (GB) invariant. It is known that G is topological invariant in four dimensions while it may lead to number of interesting cosmological effects in higher-dimensional brane-world approach (for review, see [12]). It naturally appears in the low energy effective action from string/M-theory (for recent discussion of late-time cosmology in stringy gravity with GB term, see [13]). As we demonstrate below, modified $f(G)$ gravity passes solar system tests for reasonable choice of the function f. Moreover, it is shown that such modified GB gravity may describe late-time (effective quintessence, phantom or cosmological constant) acceleration of the universe. For quite large class of functions f it is possible to describe the transition from deceleration to acceleration or from non-phantom phase to phantom phase in the late universe within such theory. Thus, modified GB gravity represents quite interesting gravitational alternative for dark energy with more freedom if compare with $f(R)$ gravity.

2. Let us start from the following action:

$$S=\int d^4x\,\sqrt{-g}\left(\frac{1}{2\kappa^2}R+f(G)\right). \tag{1}$$

Here G is the GB invariant: $G=R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma}$. By introducing two auxiliary fields A and B, one may rewrite the action (1) as

$$S=\int d^4x\,\sqrt{-g}\left(\frac{1}{2\kappa^2}R+B(G-A)+f(A)\right). \tag{2}$$

Varying over B, it follows $A=G$. Using it in (2), the action (1) is recovered. On the other hand, by the variation over A in (2), one gets $B=f'(A)$. Hence,

$$S=\int d^4x\,\sqrt{-g}\times\left(\frac{1}{2\kappa^2}R+f'(A)G-Af'(A)+f(A)\right). \tag{3}$$

The scalar is not dynamical, it has no kinetic term and is introduced for simplicity. Varying over A, the relation $A=G$ is obtained again.

The spatially-flat FRW universe metric is chosen as

$$ds^2=-dt^2+a(t)^2\sum_{i=1}^{3}\left(dx^i\right)^2. \tag{4}$$

The first FRW equation has the following form:

$$0=-\frac{3}{\kappa^2}H^2+Af'(A)-f(A)-24\dot{A}f''(A)H^3. \tag{5}$$

Here the Hubble rate H is defined by $H\equiv\dot{a}/a$. For (4), GB invariant G (A) has the following form:

$$G=A=24\left(\dot{H}H^2+H^4\right). \tag{6}$$

In general, Eq. (5) has de Sitter universe solution where H and therefore $A=G$ are constants. If $H=H_0$ with constant H_0, Eq. (5) looks as:

$$0=-\frac{3}{\kappa^2}H_0^2+24H_0^4f'\left(24H_0^4\right)-f\left(24H_0^4\right). \tag{7}$$

For large number of choices of the function f, Eq. (7) has a non-trivial ($H_0\neq 0$) real solution for H_0 (de Sitter universe). Hence, such de Sitter solution may be applied for description of the early-time inflationary as well as late-time accelerating universe.

Let us check now how modified GB gravity passes the solar system tests. The GB correction to the Newton law may be found from the coupling matter to the action (1). Varying over $g_{\mu\nu}$, we obtain

$$\begin{aligned}
0={}&\frac{1}{2\kappa^2}\left(-R^{\mu\nu}+\frac{1}{2}g^{\mu\nu}R\right)+T^{\mu\nu}+\frac{1}{2}g^{\mu\nu}f(G)\\
&-2f'(G)RR^{\mu\nu}+4f'(G)R^{\mu}{}_{\rho}R^{\nu\rho}\\
&-2f'(G)R^{\mu\rho\sigma\tau}R^{\nu}{}_{\rho\sigma\tau}-4f'(G)R^{\mu\rho\sigma\nu}R_{\rho\sigma}\\
&+2\left(\nabla^\mu\nabla^\nu f'(G)\right)R-2g^{\mu\nu}\left(\nabla^2f'(G)\right)R\\
&-4\left(\nabla_\rho\nabla^\mu f'(G)\right)R^{\nu\rho}-4\left(\nabla_\rho\nabla^\nu f'(G)\right)R^{\mu\rho}\\
&+4\left(\nabla^2f'(G)\right)R^{\mu\nu}+4g^{\mu\nu}\left(\nabla_\rho\nabla_\sigma f'(G)\right)R^{\rho\sigma}\\
&-4\left(\nabla_\rho\nabla_\sigma f'(G)\right)R^{\mu\rho\nu\sigma}.
\end{aligned} \tag{8}$$

Here $T^{\mu\nu}$ is matter EMT. In the expression (8), the third derivative $f'''(G)$ is included as $\nabla^2f'(G)=f'''(G)\nabla^2G+f''(G)\nabla^\mu G\nabla_\mu G$, for example. In Eq. (5) corresponding to the first FRW equation, however, the terms including f''' do not appear. When $T^{\mu\nu}=0$, the (t,t)-component of Eq. (8) reproduces