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Numerical study of the enlarged O(5) symmetry of the 3D antiferromagnetic RP^2 spin model

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Abstract

We investigate by means of Monte Carlo simulation and finite-size scaling analysis the critical properties of the three-dimensional O(5) non-linear σ model and of the antiferromagnetic $\mathbf{R}P^2$ model, both of them regularized on a lattice. High accuracy estimates are obtained for the critical exponents, universal dimensionless quantities and critical couplings. It is concluded that both models belong to the same universality class, provided that rather non-standard identifications are made for the momentum-space propagator of the $\mathbf{R}P^2$ model. We have also investigated the phase diagram of the $\mathbf{R}P^2$ model extended by a second-neighbor interaction. A rich phase diagram is found, where most of the phase transitions are of the first order.

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1. Introduction

Universality is sometimes expressed in a somehow defectively simple way: some critical properties (the universal ones) of a system are given by space dimensionality and the local properties (i.e., near the identity element) of the coset space $\mathcal{G}/\mathcal{H}$, where $\mathcal{G}$ is the symmetry group of the high-temperature phase and $\mathcal{H}$ is the remaining symmetry group of the broken phase (low temperature). As we shall discuss, the subtle point making the above statement not straightforward to use, is that $\mathcal{G}$ needs not to be the symmetry group of the microscopic Hamiltonian, but that of the coarse-grained fixed-point action.

On the spirit of the above statement, some time ago [1] a seemingly complete classification was obtained of the universality classes of three-dimensional

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systems where $\mathcal{G} = \mathrm{O}(3)$. In this picture, a phase transition of a vector model, with O(3) global symmetry and with an O(2) low-temperature phase symmetry, in three dimensions must belong to the $\mathrm{O}(3)/\mathrm{O}(2)$ scheme of symmetry breaking (classical Heisenberg model). In addition, if $\mathcal{H} = \mathrm{O}(1) = Z_2$ is the remaining symmetry, the corresponding scheme should be $\mathrm{O}(4)/\mathrm{O}(3)$ which is locally isomorphic to $\mathrm{O}(3)/\mathrm{O}(1)$.[1] This classification has been challenged by the chiral models [2]. However, the situation is still hotly debated: some authors believe that the chiral transitions are weakly first-order [3], while others claim [4] that the chiral universality class exists, implying the relevance of the global properties of $\mathcal{G}/\mathcal{H}$.

In this Letter, we shall consider the three-dimensional antiferromagnetic (AFM) $\mathbf{R}\mathrm{P}^2$ model [5–8], a model displaying a second-order phase transition and escaping from the previously expressed paradigm. It is worth recalling [9,10] that one of the phase transitions found in models for colossal magnetoresistance oxides [11] belongs to the universality class of the AFM $\mathbf{R}\mathrm{P}^2$ model. The microscopic Hamiltonian of this model has a global O(3) symmetry group, while the low-temperature phase has, at least, a remaining O(2) symmetry [9]. We will show here that the model belongs to the universality class of the three-dimensional O(5) non-linear σ model. Some ground for this arises from a hand-waving argument, suggested to us by one of the referees of Ref. [9] (see below).

The universality class of the three-dimensional O(5) non-linear σ model has received less attention that $\mathrm{O}(N)$ models with $N = 0, 1, 2, 3$ and 4. In spite of that, it has been recently argued that O(5) could be relevant for the high-temperature superconducting cuprates [12]. Nevertheless, perturbative field-theoretic methods have been used to estimate the critical exponents [13–16]. From the numerical side, only a rather unconvincing Monte Carlo simulation [17] was available until very recently. Fortunately, there has been a recent, much more careful study [18]. Yet, the scope of Ref. [18] was to determine whether an interaction explicitly degrading the O(5) symmetry to an $\mathrm{O}(3) \oplus \mathrm{O}(2)$ group was relevant in the renormalization-group sense. To that end, those authors concentrated in producing extremely accurate data on small lattices.

Our purpose is to study in greater detail the critical properties of the three-dimensional O(5) non-linear σ model, and of the AFM $\mathbf{R}\mathrm{P}^2$ model. We improve over previous studies of both models, obtaining more accurate estimates for critical exponents, universal dimensionless quantities and non-universal critical couplings. As symmetries play such a prominent role, we will also explore the possibilities of changing those of the low-temperature phase by adding a second-neighbors coupling to the Hamiltonian of the AFM $\mathbf{R}\mathrm{P}^2$ model.

2. The models

We are considering a system of N-component normalized spins $\{\vec{v}_i\}$ placed in a three-dimensional simple cubic lattice of size L with periodic boundary conditions. The actions of our lattice systems are

$$\mathcal{S}^{\mathrm{O}(N)} = -\beta \sum_{\langle i,j\rangle} (\vec{v}_i \cdot \vec{v}_j),$$
$$\mathcal{S}^{\mathbf{R}\mathrm{P}^{N-1}} = -\beta \sum_{\langle i,j\rangle} (\vec{v}_i \cdot \vec{v}_j)^2, \tag{1}$$

where the sums are extended to all pairs of nearest neighbors. Our sign convention is fixed by the partition function:

$$Z = \int \prod_i d\vec{v}_i \, \mathrm{e}^{-\mathcal{S}}, \tag{2}$$

$d\vec{v}$ being the rotationally invariant measure over the N-dimensional unit sphere.

To construct observables, in addition to the vector field $\vec{v}_i$, we consider the (traceless) tensorial field

$$\tau_i^{\alpha\beta} = v_i^\alpha v_i^\beta - \frac{1}{N}\delta^{\alpha\beta}, \quad \alpha, \beta = 1, \ldots, N. \tag{3}$$

The interesting quantities related with the order parameters can be constructed in terms of the Fourier transforms of the fields ($f_i = \vec{v}_i, \tau_i$)

$$\hat{f}(\boldsymbol{p}) = \frac{1}{L^3} \sum_i \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}_i} f_i. \tag{4}$$

For $\mathbf{R}\mathrm{P}^{N-1}$ models, the local gauge invariance $\vec{v}_i \to -\vec{v}_i$ implies that the relevant observables are

[1] This statement assumes that the *global* properties of the coset $\mathcal{G}/\mathcal{H}$ are irrelevant, only the local properties matter.