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# Ground state baryons in $\tilde{U}(12)$ scheme

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**Abstract**

The properties of ground-state baryons of light-quarks are investigated in $\tilde{U}(12)$-classification scheme of hadrons, recently proposed by us. In $\tilde{U}(12)$, in addition to the ordinary **56** of $SU(6)_{\mathrm{SF}}$, the existence of the extra **56** with positive-parity and **70** with negative-parity appear as ground-states in lower mass region. The $N(1440)$, $\Lambda(1600)$ and $\Sigma(1660)$ have the plausible properties of masses and strong decay widths as the flavor-octet of extra **56**, while the ordinary radially-excited octet states are expected not to be observed as resonances because of their large predicted widths of the decays to extra **70** baryons. The extra **70** baryons are not observed directly as resonances for the same reason, except for only the $\Lambda(1406)$, of which properties are consistently explained through the singlet–octet mixing. The baryon properties in lower mass regions are consistently explained in $\tilde{U}(12)$ scheme.



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## 1. Introduction

The spectroscopy of light-quark $qqq$ baryons is longstanding problem of hadron physics. The non-relativistic quark model (NRQM) successfully explain the properties of ground state **56**-multiplet of $SU(6)$ spin-flavor symmetry. On the other hand, NRQM predicts the negative-parity states as the next low-lying states from orbital excitation, while the experiments show the clear evidence of the Roper resonance $N(1440)$ of the second nucleon state with positive-parity. The situation is similar for $\Delta$, $\Lambda$ and $\Sigma$ systems. The positive-parity $\Delta(1600)$, $\Lambda(1600)$ and $\Sigma(1660)$ have too light masses to be naturally assigned as radially excited states in NRQM. The mass of the negative parity $\Lambda(1405)$ is also too light to be assigned as the first excited **70**-multiplet in NRQM.

Recently [1], we have proposed a covariant level-classification scheme of hadrons based on $\tilde{U}(12)$ group. In this scheme, the squared-mass spectra of hadrons including light constituent quarks are classi-

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Table 1
Flavor-spinor WF of ground-state $qqq$-baryon. The indices $S,\alpha(\beta),A$ represent the completely symmetric, partially symmetric (anti-symmetric) and completely anti-symmetric WFs of permutation group

| $SU(6)_{\mathrm{SF}}$ | Spin-flavor wave function | $B^{\mathcal{P}}$ | $SU(3)_F$ |
|---|---|---|---|
| $\mathbf{56}_E$ | $\lvert\rho\rangle_S\lvert F\sigma\rangle_S=\lvert\rho,\frac{3}{2}\rangle_S\lvert F\rangle_S\lvert\sigma\rangle_S$ | $\Delta^{\oplus}_{3/2}$ | $\mathbf{10}$ |
| | $\lvert\rho,\frac{3}{2}\rangle_S\frac{1}{\sqrt{2}}(\lvert F\rangle_\alpha\lvert\sigma\rangle_\alpha+\lvert F\rangle_\beta\lvert\sigma\rangle_\beta)$ | $N^{\oplus}_{1/2}$ | $\mathbf{8}$ |
| $\mathbf{70}_G$ | $\frac{1}{\sqrt{2}}(\lvert\rho\rangle_\alpha\lvert F\sigma\rangle_\alpha+\lvert\rho\rangle_\beta\lvert F\sigma\rangle_\beta)$, $\quad\lvert F\sigma\rangle_{\alpha(\beta)}=\lvert F\rangle_S\lvert\sigma\rangle_{\alpha(\beta)}$ | $\Delta^{\ominus}_{1/2}$ | $\mathbf{10}$ |
| | $\lvert F\rangle_{\alpha(\beta)}\lvert\sigma\rangle_S$ | $N^{\ominus}_{3/2}$ | $\mathbf{8}$ |
| | $\lvert F\sigma\rangle_{\alpha(\beta)}=\frac{1}{\sqrt{2}}(\mp\lvert F\rangle_\alpha\lvert\sigma\rangle_{\alpha(\beta)}+\lvert F\rangle_\beta\lvert\sigma\rangle_{\beta(\alpha)})$ | $N^{\ominus}_{1/2}$ | $\mathbf{8}$ |
| | $\lvert F\rangle_A\lvert\rho\sigma\rangle_A=\lvert F\rangle_A\frac{1}{\sqrt{2}}(-\lvert\rho,\frac{1}{2}\rangle_\alpha\lvert\sigma\rangle_\beta+\lvert\rho,\frac{1}{2}\rangle_\beta\lvert\sigma\rangle_\alpha)$ | $\Lambda^{\ominus}_{1/2}$ | $\mathbf{1}$ |
| $\mathbf{56}_F$ | $\lvert\rho\rangle_S\lvert F\sigma\rangle_S=\lvert\rho,-\frac{1}{2}\rangle_S\lvert F\rangle_S\lvert\sigma\rangle_S$ | $\Delta^{\oplus}_{3/2}$ | $\mathbf{10}$ |
| | $\lvert\rho,-\frac{1}{2}\rangle_S\frac{1}{\sqrt{2}}(\lvert F\rangle_\alpha\lvert\sigma\rangle_\alpha+\lvert F\rangle_\beta\lvert\sigma\rangle_\beta)$ | $N^{\oplus}_{1/2}$ | $\mathbf{8}$ |

fied as the representation of $\tilde{U}(12)$ spin-flavor group. In this scheme we have introduced the expansion bases of spinor wave functions (WF) of composite hadrons. Each spinor index corresponding to light quark freedom is expanded by free Dirac spinors $u_{r,s}(v_\mu)$.

$$u_{+,s}(v_\mu)=\begin{pmatrix}\mathrm{ch}\,\theta\,\chi^{(s)}\\ \mathrm{sh}\,\theta\,\boldsymbol{n}\cdot\boldsymbol{\sigma}\,\chi^{(s)}\end{pmatrix},$$
$$u_{-,s}(v_\mu)=\begin{pmatrix}\mathrm{sh}\,\theta\,\boldsymbol{n}\cdot\boldsymbol{\sigma}\,\chi^{(s)}\\ \mathrm{ch}\,\theta\,\chi^{(s)}\end{pmatrix},\tag{1}$$

where $\mathrm{ch}\,\theta,\mathrm{sh}\,\theta=\sqrt{(\omega\pm1)/2}$ and the $v_\mu\equiv P_\mu/M=(\boldsymbol{n}\omega_3;i\omega)_\mu$ is the four velocity of the relevant hadron. This method is invented for keeping the Lorentz-covariance of the composite system. (Concerning the $\tilde{U}(12)$ scheme and its group theoretical arguments, see our Ref. [2].) We should note both $u_+$ and $u_-$ is necessary for expansion bases of quark spinor index. They are called urciton spinors for historical reason [3].

The $u_{+,s}$ corresponds to the ordinary spinor freedom appearing in NRQM, while the $u_{-,s}$ represents the relativistic effect. In the $\tilde{U}(12)$-classification scheme, the $u_{-,s}$ is supposed to appear as new degrees of freedom for light constituents, being independent of $u_{+,s}$. The freedom corresponding $r$ index of $u_{r,s}$ is called $\rho$-spin, while the ordinary Pauli-spin freedom described by $\chi^{(s)}$ is called $\sigma$-spin, where $\rho\times\sigma$ corresponds to the $\rho\sigma$ decomposition of Dirac $\gamma$ matrices. The index $s$ of $u_{r,s}$ represents the eigenvalue of $\sigma_3$, while the $r$ does the eigenvalue of $\rho_3$ at the hadron rest frame, where $v_\mu=v_{0\mu}=(\mathbf{0};i)_\mu$.

Because of this extra $SU(2)$ spin freedom, the $SU(6)_{\mathrm{SF}}$ is extended to $\tilde{U}(12)$, or more precisely $U(12)_{\mathrm{stat}}$ at the hadron rest frame, as $U(12)_{\mathrm{stat}}\supset SU(2)_\rho\times SU(2)_\sigma\times SU(3)_F$ (see, Ref. [2]). The ground-state $qqq$ baryons and antibaryons are assigned as the completely symmetric $(\mathbf{12}\times\mathbf{12}\times\mathbf{12})_{\mathrm{sym}}=\mathbf{364}$ representation of $U(12)_{\mathrm{stat}}$ or $\tilde{U}(12)$. The corresponding flavor-spinor WFs $\Phi_{ABC}(v_\mu)$ are represented by the direct product of flavor and spinor WFs as

$$\Phi_{ABC}(v)\sim\lvert F\rangle_{abc}u_\alpha(v)u_\beta(v)u_\gamma(v),\tag{2}$$

where $A=(a,\alpha)$, etc., denote the (flavor, spinor) indices. $\lvert F\rangle(u(v))$ represents the flavor (spinor) WF. At the rest frame of hadron, the $u_\alpha(v)$'s are decomposed as $\rho$ and $\sigma$ spin WFs denoted as $\lvert\rho,\rho_3/2\rangle$ and $\lvert\sigma_3/2\rangle$, respectively. The ground-state baryons in $\tilde{U}(12)$ are classified as $\mathbf{364}/2=\mathbf{56}_E+\mathbf{70}_G+\mathbf{56}_F$ in terms of $SU(6)_{\mathrm{SF}}$. The explicit forms of WFs are given in Table 1.

The $\mathbf{56}_E$ has all the three spinor indices with positive $\rho_3$ and $\rho$-spin WF is given by the completely symmetric $\lvert\rho,\frac{3}{2}\rangle_S$. This freedom corresponds to the ones appearing in NRQM, and the corresponding states are called Pauli-states (or paulons). The $\mathbf{56}_E$ includes the $N(939)$-octet and $\Delta(1232)$-decouplet. While $\mathbf{70}_G(\mathbf{56}_F)$ have one index (two indices) with negative $\rho_3$ and are described by the $\rho$-spin WFs