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Thermodynamic behavior of fuzzy membranes in pp-wave matrix model

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Abstract

We discuss a two-body interaction of membrane fuzzy spheres in a pp-wave matrix model at finite temperature by considering the system that a fuzzy sphere rotates with a constant radius r around the other one sitting at the origin in the $SO(6)$ symmetric space. This system of two fuzzy spheres is supersymmetric at zero temperature and there is no interaction between them. Once the system is coupled to the heat bath, supersymmetries are completely broken and non-trivial interaction appears. We numerically show that the potential between fuzzy spheres is attractive and so the rotating fuzzy sphere tends to fall into the origin. The analytic formula of the free energy is also evaluated in the large N limit. It is well approximated by a polylog function.

Keywords: pp-wave matrix model; Fuzzy sphere; Giant graviton; Thermodynamics

1. Introduction and summary

The basic degrees of freedom of string theory and M-theory are fully encoded in matrix models [1–3]. The matrix models are believed to give non-perturbative formulations of string theory and M-theory. In particular, the BFSS matrix model is supersymmetric matrix quantum mechanics. It is believed to describe a discrete light-cone quantization of M-theory. It also describes the low-energy dynamics of N D0-branes of type IIA superstring theory [4]. In addition it is a matrix regularization of the light-cone action for the supermembrane in eleven dimensions [5].

Matrix model thermodynamics is also interesting in relation to black hole physics. One motivation for understanding the behavior of the BFSS matrix model at finite temperature comes from the conjecture that their finite

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temperature states are related to black hole states of type IIA supergravity [6,7]. This idea was studied in a series of papers by Kabat et al. [8]. The BFSS matrix model at finite temperature is also investigated in [9]. Some features are discussed in [10] (for a review of brane thermodynamics, see [11]).

By the way, a matrix model on a pp-wave background was proposed by Berenstein–Maldacena–Nastase (BMN) [12], and it has been intensively studied. The background of this matrix model is given by the maximally supersymmetric pp-wave background [13]:

$$\begin{aligned} ds^2 &= -2\,dx^+\,dx^- - \left(\sum_{i=1}^{3}\left(\frac{\mu}{3}\right)^2 (x^i)^2 + \sum_{a=4}^{6}\left(\frac{\mu}{6}\right)^2 (x^a)^2\right)(dx^+)^2 + \sum_{I=1}^{9}(dx^I)^2, \\ F_{+123} &= \mu. \end{aligned} \tag{1.1}$$

The action of the matrix model on this background S_{pp} consists of two parts as follows[1]:

$$S_{\text{pp}} = S_{\text{flat}} + S_\mu, \tag{1.2}$$

$$\begin{aligned} S_{\text{flat}} &= \int dt\,\mathrm{Tr}\left[\frac{1}{2R}D_t X^I D_t X^I + \frac{R}{4}\left(\left[X^I, X^J\right]\right)^2 + i\Theta^\dagger D_t \Theta - R\Theta^\dagger \gamma^I\left[\Theta, X^I\right]\right], \\ S_\mu &= \int dt\,\mathrm{Tr}\left[-\frac{1}{2R}\left(\frac{\mu}{3}\right)^2 (X^i)^2 - \frac{1}{2R}\left(\frac{\mu}{6}\right)^2 (X^a)^2 - i\frac{\mu}{3}\epsilon^{ijk}X^i X^j X^k - i\frac{\mu}{4}\Theta^\dagger\gamma^{123}\Theta\right], \end{aligned} \tag{1.3}$$

where the indices of the transverse nine-dimensional space are $I, J = 1, \ldots, 9$ and R is the radius of the circle compactified along x^-. All degrees of freedom are $N \times N$ Hermitian matrices and the covariant derivative D_t with the gauge field A is defined by $D_t = \partial_t - i[A,\ ]$. This matrix model is closely related to the supermembrane theory on the pp-wave background via the matrix regularization [5] (for works in this direction see [14,15]). This matrix model has a supersymmetric fuzzy sphere solution (which is called "giant graviton") due to the Myers effects [16], because the constant 4-form flux is equipped with. This fuzzy sphere solution is given by

$$X^i_{\text{sphere}} = \frac{\mu}{3}J^i, \tag{1.4}$$

where J^i satisfies the $SU(2)$ algebra $[J^i, J^j] = i\epsilon^{ijk}J^k$. The classical energy of this solution is zero and hence the fuzzy sphere can appear in classical vacua without loss of energy. Namely, the classical vacua of the pp-wave matrix model are enriched with fuzzy spheres, and it may be interesting to look deeper into the dynamics of fuzzy sphere solution. Then the fuzzy sphere solution X^i_{sphere} preserves the full 16 dynamical supersymmetries of the pp-wave and hence is 1/2-BPS object. We note that actually there is another fuzzy sphere solution of the form $\frac{\mu}{6}J^i$ but the solution does not have quantum stability and is thus non-BPS object [17] (for other classical solutions, see [18–21]). In [22] the interaction potential of membrane fuzzy spheres was shown to be the $1/r^7$-type.

In this Letter we discuss a thermal interaction between fuzzy membrane solutions in the pp-wave matrix model. The two-body interaction of fuzzy spheres at zero temperature was studied in our previous work [23] (containing the extension of the work [24]) by considering the system that a fuzzy sphere rotates with a constant radius r around the other one sitting at the origin in the transverse six-dimensional space (Fig. 1). In the case of zero temperature the interaction potential is zero basically because of the existence of the remaining supersymmetries. In the finite temperature case, however, non-trivial interaction should appear since the supersymmetries are completely broken due to the thermal effect. Here, we will be interested in the evaluation of the thermal potential at finite temperature by heating the system depicted in Fig. 1. Firstly we consider the exact one-loop free energy by using the $\mu \to \infty$ limit. In this computation we use the spectrum around the two-body system which has been obtained in [23]. By numerically plotting the free energy, we can see that the rotating fuzzy sphere tends to approach to the static fuzzy

[1] Hereafter we will rescale the gauge field and parameters as $A \to RA$, $t \to t/R$, $\mu \to R\mu$.