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Physics Letters B 621 (2005) 109–118
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Recursive renormalization of the singlet one-pion-exchange plus point-like interactions

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Received 3 January 2005; received in revised form 3 April 2005; accepted 16 June 2005

Available online 23 June 2005

Editor: J.-P. Blaizot

Abstract

The subtracted kernel approach is shown to be a powerful method to be implemented recursively in scattering equations with regular plus point-like interactions. The advantages of the method allows one to recursively renormalize the potentials, with higher derivatives of the Dirac-delta, improving previous results. The applicability of the method is verified in the calculation of the $^1S_0$ nucleon–nucleon phase-shifts, when considering a potential with one-pion-exchange plus a contact interaction and its derivatives. The $^1S_0$ renormalization parameters are fitted to the data. The method can in principle be extended to any derivative order of the contact interaction, to higher partial waves and to coupled channels.

PACS: 03.65.Ca; 11.10.Hi; 05.10.Cc; 03.65.Nk

Keywords: Renormalization; Renormalization group; Nonrelativistic scattering theory

1. Introduction

The pioneering work of Weinberg [1] launched the basis for the effective field theory (EFT) of nuclear forces starting from the expansion of an effective chiral Lagrangian. It gives a nucleon–nucleon (NN) interaction which is, in leading order, the one-pion-exchange potential (OPEP) plus a Dirac-delta. The program of applying effective field methods to the NN system was pursued by many authors with significative results in few-nucleon systems

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2005.06.044

(see, e.g., [2]). In a more general context of few-body systems, short range interactions have many applications which are discussed in detail in Ref. [3].

More recently, the authors of [4,5] treat the one-pion-exchange potential (OPEP) plus derivative Dirac-delta interactions using dimensional and boundary condition regularizations, respectively. In particular, when comparing the theoretical approach with the data [6], it was shown in Ref. [7] that the leading-order interaction, OPEP plus a Dirac-delta, renormalized using subtracted scattering equations, dominates the results obtained for the $^3S_1$–$^3D_1$ phase-shifts and mixing parameter. However, considering only the leading-order term, the results obtained for the $^1S_0$ phase shift are not satisfactory. This is an evidence that higher-order terms in the effective interaction are important in this channel (see, e.g., Refs. [4,5]).

The fit of the $^1S_0$ phase-shift up to laboratory momentum $p_{\rm Lab} \sim 300$ MeV/c requires an effective NN interaction with the addition of a term with second-order derivatives of the Dirac-delta. In the relative momentum space, we have

$$\langle \vec{p}|V|\vec{p}\rangle = \langle \vec{p}'|V_\pi^s|\vec{p}\rangle + \sum_{i,j=0}^{1} \lambda_{ij}\, p'^{2i} p^{2j}, \tag{1}$$

where the λ's are unregulated strengths and $\langle \vec{p}'|V_\pi^s|\vec{p}\rangle$ is a matrix element of the one-pion-exchange potential. The motivation of the second term of (1) is to simulate effects of heavy particle exchanges by a sum of a Dirac-delta interaction and its derivatives, while keeping the OPE as the long range part of the interaction. In the renormalization method described in [7], we include λ_{11}, as the method is based on a kernel-subtraction procedure that generates terms of the type $p'^2 p^2$ in the scattering matrix elements even when one considers only p^2 and p'^2 in the interaction. In fact, the method also generates higher-order derivatives terms when OPE is considered. In the Weinberg counting rule scheme, the derivative contact interactions we are considering comes along with the two-pion-exchange (TPE) [1,8]. For a recent calculation in the NN system considering TPE see, e.g., Ref. [9]. As an exploratory calculation, in the present work we consider only the contact terms, leaving TPE for a future work.

In the scattering equation, the effective bare potential (1) produces integrals that diverge as much as p^5. Therefore it is necessary at least three subtractions in the kernel of the Lippmann–Schwinger (LS) equation, since each subtraction introduces a factor of p^{-2}. Differently from the recent works [4,5], we implement the method of subtracted scattering equations [10] to deal with this problem. The subtraction method has also been shown to be practical in providing renormalization group invariant solutions for three-body scattering equations with contact interactions [11–13], and also proved to be useful in describing the halo structure of weakly bound exotic light nuclei [14].

The one-subtracted scattering equation used in our previous work [7] was generalized in [10] to allow any order of subtraction, permitting the inclusion of derivatives of the contact interaction in the effective two-body potential. The driving term of the n-subtracted LS equation is constructed recursively, renormalizing the model at each subtraction order, while keeping renormalization group invariance of the approach (for details see [10]).

In the present Letter, we obtain the $^1S_0$ nucleon–nucleon amplitude from the effective interaction (1), using a three-times subtracted scattering equation, as demanded by the higher divergent term of the type p^5. We perform an analysis of the physical contribution coming from each order term in the recursive order-by-order renormalization procedure. It this way we access their significance in the parameterization of the effective interaction to obtain the desired observables.

We should note that the contact interactions that we are considering are obviously meaningless without regularization and renormalization due to the generated ultraviolet divergences in the Lippmann–Schwinger equation.

This Letter is organized as follows. In Section 2, we discuss the subtraction method applied to OPEP. In Section 3, we present the main formulas of the recursive subtraction method to treat the scattering equation, as well as our strategy to solve it. In Section 4, we show numerical results for the singlet phase-shift with our conclusion.