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Physics Letters B 619 (2005) 193–200
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

The fate of (phantom) dark energy universe with string curvature corrections

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Received 26 April 2005; received in revised form 7 June 2005; accepted 8 June 2005

Available online 16 June 2005

Editor: M. Cvetič

Abstract

We study the evolution of (phantom) dark energy universe by taking into account the higher-order string corrections to Einstein–Hilbert action with fixed dilaton and modulus fields. While the presence of a cosmological constant gives stable de Sitter fixed points in the cases of heterotic and bosonic strings, no stable de Sitter solutions exist when a phantom fluid is present. We find that the universe can exhibit a Big Crunch singularity with a finite time for type II string, whereas it reaches a Big Rip singularity for heterotic and bosonic strings. Thus the fate of dark energy universe crucially depends upon the type of string theory under consideration.

PACS: 98.70.Vc

1. Introduction

Recent observations suggest that the current universe is dominated by dark energy responsible for an accelerated expansion [1]. The equation of state parameter w for dark energy lies in a narrow region around $w = -1$ and may even be smaller than -1 [2]. When w is less than -1, dubbed as phantom dark energy, the universe ends up with a Big Rip singularity [3,4] which is characterized by the divergence of curvature of the universe after a finite interval of time (see Refs. [5,6]).

The energy scale may grow up to the Planck scale in the presence of phantom dark energy. This means that higher-order curvature or quantum corrections can be important around the Big Rip. For example, quantum corrections coming from conformal anomaly are taken into account in Refs. [7] for dark energy dynamics. It was found that such corrections can moderate

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0370-2693/$ – see front matter © 2005 Published by Elsevier B.V.
doi:10.1016/j.physletb.2005.06.017

the singularity by providing a negative energy density [8]. Thus it is important to implement quantum effects in order to predict the final fate of the universe.

In low-energy effective string theory there exist higher-curvature corrections to the usual scalar curvature term. The leading quadratic correction corresponds to the product of dilaton/modulus and Gauss–Bonnet (GB) curvature invariant [9]. The GB term is topologically invariant in four dimensions and hence does not contribute to dynamical equations of motion if the dilaton/modulus field is constant [10]. Meanwhile it affects the cosmological dynamics in presence of dynamically evolving dilaton and modulus fields. The possible effects of the GB term for early universe cosmology and black hole physics were investigated in Refs. [11,12]. Lately the GB correction was applied to the study of cosmological dynamics of dark energy [13].

When the dilaton and modulus are fixed, it is important to implement third and next-order string curvature corrections [10]. This can change the resulting cosmological dynamics drastically as it happens in the context of inflation [14] and black holes [15]. The goal of the present Letter is to study the effect of next-to-leading order string corrections to the cosmological dynamics around the Big Rip singularity with an assumption that the dilaton and the modulus are stabilized. We would also investigate the existence and the stability of de Sitter solutions in the presence of a cosmological constant. We shall consider three types of string corrections and study the fate of the universe accordingly.

2. Evolution equations

Let us consider the Einstein–Hilbert action in low-energy effective string theory:

$$\mathcal{S} = \int d^D x \sqrt{-g}\,[R + \mathcal{L}_c + \cdots], \tag{1}$$

where R is the scalar curvature and $\mathcal{L}_c$ is the string correction which is given by [10]

$$\mathcal{L}_c = c_1\alpha' e^{-2\phi}\mathcal{L}_2 + c_2\alpha'^2 e^{-4\phi}\mathcal{L}_3 + c_3\alpha'^3 e^{-6\phi}\mathcal{L}_4, \tag{2}$$

where α' is the string expansion parameter, ϕ is the dilaton field, and

$$\mathcal{L}_2 = \Omega_2, \tag{3}$$

$$\mathcal{L}_3 = 2\Omega_3 + R^{\mu\nu}_{\alpha\beta} R^{\alpha\beta}_{\lambda\rho} R^{\lambda\rho}_{\mu\nu}, \tag{4}$$

$$\mathcal{L}_4 = \mathcal{L}_{41} - \delta_H \mathcal{L}_{42} - \frac{\delta_B}{2}\mathcal{L}_{43}, \tag{5}$$

with

$$\Omega_2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, \tag{6}$$

$$\begin{aligned}\Omega_3 = {} & R^{\mu\nu}_{\alpha\beta} R^{\alpha\beta}_{\lambda\rho} R^{\lambda\rho}_{\mu\nu} - 2R^{\mu\nu}_{\alpha\beta} R^{\lambda\beta\rho}_{\nu} R^{\alpha}_{\rho\mu\lambda} \\ & + \frac{3}{4} R R^2_{\mu\nu\alpha\beta} + 6R^{\mu\nu\alpha\beta}R_{\alpha\mu}R_{\beta\nu} \\ & + 4R^{\mu\nu}R_{\nu\alpha}R^{\alpha}_{\mu} - 6RR^2_{\alpha\beta} + \frac{R^3}{4},\end{aligned} \tag{7}$$

$$\mathcal{L}_{41} = \zeta(3) R_{\mu\nu\rho\sigma}R^{\alpha\nu\rho\beta}\big(R^{\mu\gamma}_{\delta\beta}R^{\delta\sigma}_{\alpha\gamma} - 2R^{\mu\gamma}_{\delta\alpha}R^{\delta\sigma}_{\beta\gamma}\big), \tag{8}$$

$$\begin{aligned}\mathcal{L}_{42} = {} & \frac{1}{8}\big(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\big)^2 + \frac{1}{4}R^{\gamma\delta}_{\mu\nu}R^{\rho\sigma}_{\gamma\delta}R^{\alpha\beta}_{\rho\sigma}R^{\mu\nu}_{\alpha\beta} \\ & - \frac{1}{2}R^{\alpha\beta}_{\mu\nu}R^{\rho\sigma}_{\alpha\beta}R^{\mu}_{\sigma\gamma\delta}R^{\nu\gamma\delta}_{\rho} - R^{\alpha\beta}_{\mu\nu}R^{\rho\nu}_{\alpha\beta}R^{\gamma\delta}_{\rho\sigma}R^{\mu\sigma}_{\gamma\delta},\end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{L}_{43} = {} & \big(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\big)^2 - 10R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\sigma}R_{\sigma\gamma\delta\rho}R^{\beta\gamma\delta\rho} \\ & - R_{\mu\nu\alpha\beta}R^{\mu\nu\rho}_{\sigma}R^{\beta\sigma\gamma\delta}R^{\alpha}_{\delta\gamma\rho}.\end{aligned} \tag{10}$$

Here one has $\delta_{H(B)} = 1$ for heterotic (bosonic) string and zero otherwise. The Gauss–Bonnet term, Ω_2, as well as the Euler density, Ω_3, does not contribute to the background equation of motion for $D = 4$ unless the dilaton is dynamically evolving. The coefficients (c_1, c_2, c_3) are different depending on string theories [10]. We have $(c_1, c_2, c_3) = (0, 0, 1/8), (1/8, 0, 1/8), (1/4, 1/48, 1/8)$ for type II, heterotic, and bosonic strings, respectively. In the case of type II string with $D = 4$, for example, only the $\mathcal{L}_{41}$ term affects the dynamical evolution of the system.

We shall consider the flat Friedmann–Robertson–Walker metric with a lapse function $N(t)$:

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \sum_{i=1}^{d} \big(dx^i\big)^2, \tag{11}$$

where $d = D - 1$. The Ricci tensors under this metric are given in Appendix A. In what follows we shall consider the case of $D = 4$ under the assumption that the modulus field which corresponds to the radius of