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Noncompact KK theory of gravity: Stochastic treatment for a nonperturbative inflaton field in a de Sitter expansion

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Abstract

We study a stochastic formalism for a nonperturbative treatment of the inflaton field in the framework of a noncompact Kaluza–Klein (KK) theory during an inflationary (de Sitter) expansion, without the slow-roll approximation.

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1. Introduction

Stochastic inflation model is one of the very few that solves almost all of the well-known cosmological problems. Since the differential microwave radiometer (DMR) mounted on the Cosmic Background Explorer satellite (COBE) first detected temperature anisotropies in the cosmic background radiation (CBR), we have the possibility to directly probe the initial density perturbation. The fact that the resulting energy density fluctuations ($\frac{\delta\rho}{\rho} \approx 10^{-5}$) fit the scaling spectrum predicted by the inflation model, suggests that they had indeed their origin in the quantum fluctuations of the "inflaton" scalar field during the inflationary era. Although in principle this problem is of a quantal nature, the fact that under certain conditions—which are made precise in [1–3]—the inflaton field can be considered as classical largely simplifies the approach, by allowing a Langevin-like stochastic treatment. The most widely accepted approach assumes that the inflationary phase is driving by a quantum scalar field φ with a potential $V(\varphi)$. Within this perspective, the stochastic inflation proposes to describe the dynamics of this quantum field on the basis of a splitting of φ in a homogeneous and an inhomogeneous components. Usually the homogeneous

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one $\phi_c(t)$ is interpreted as a classical field that arises from a coarsed-grained average over a volume larger than the observable universe, and plays the role of a global order parameter [4]. All information on scales smaller than this volume, such as the density fluctuations, is contained in the inhomogeneous component. Although this theory is widely used and accepted as general, one needs to make the approximation $\langle\rho\rangle \simeq \frac{\dot{\phi}_c^2}{2} + V(\phi_c)$ to be able to make some calculations in a linear expansion for the scalar potential $V(\varphi)$ around its classical background field $\phi_c(t)$ [3]. It was Starobinsky the first one to derive a Fokker–Planck equation for the transition probabilities $P(\phi_L, t|\phi'_L, t')$ in comoving coordinates [5] from the stochastic equation for the dynamics of the inflaton field. $P(\phi_L, t|\phi'_L, t')$ provides us with statistical information about the relative number of "domains" (metastable vacuum configurations) that having a typical value ϕ'_L of the coarse-grained inflaton field, evolve in a time interval $(t - t')$ towards a new configuration with a typical value ϕ_L.

The main aim of this work consists to make a consistent coarse-granning treatment for the scalar field dynamics on cosmological scales during the inflationary epoch. For simplicity, as an example, we shall study a de Sitter expansion for the universe, but the formalism here developed can be used to study other more realistic inflationary models. The strategy consists to starts from a 5D globally flat metric and an action for a purely kinetic quantum scalar field minimally coupled to gravity, which define a 5D vacuum state. The metric we consider can be mapped to a 5D generalized Friedmann–Robertson–Walker (FRW) on which we make a foliation by considering the fifth (spatial-like) coordinate as a constant. As a result of this foliation we obtain an effective 4D FRW metric and an effective 4D density Lagrangian in which appears an effective term which depends on the fifth coordinate and is not kinetic in 4D. This term is identified as a 4D scalar potential or source, and has an origin purely geometric. The idea that matter in four dimensions (4D) can be explained from a 5D Ricci-flat ($R_{AB} = 0$) Riemannian manifold is a consequence of the Campbell's theorem. It says that any analytic N-dimensional Riemannian manifold can be locally embedded in a $(N + 1)$-dimensional Ricci-flat manifold. This is of great importance for establishing the generality of the proposal that 4D field equations with sources can be locally embedded in 5D field equations without sources [6,7]. The advantage of this propose is that provides an exact (nonperturbative) treatment for the 4D dynamics of the inflaton field (the scalar field) with back-reaction effects included [8]. Furthermore, it is possible to make a consistent treatment for the effective 4D dynamics of the universe in other models governed by a single scalar field. In this Letter we consider a general version of the Kaluza—Klein theory in 5D, where the extra dimension is not assumed to be compactified. In other words, this means that the cylinder condition in the fifth coordinate of the original Kaluza—Klein theory is relaxed. From the mathematical viewpoint, this means that the 5D metric tensor is allowed to depend explicitly on the fifth coordinate. Without cylindricity, there is no reason to compactify the fifth dimension, so this approach is properly called noncompactified.

2. Review of the formalism

We consider an action

$$I = -\int d^4x\, d\psi \sqrt{\left|\frac{^{(5)}g}{^{(5)}g_0}\right|}\left[\frac{^{(5)}R}{16\pi G} + {}^{(5)}\mathcal{L}(\varphi, \varphi_{,A})\right], \qquad (1)$$

for a scalar field φ, which is minimally coupled to gravity. Here, $|^{(5)}g| = \psi^8 e^{6N}$ is the absolute value of the determinant for the 5D metric tensor with components g_{AB} (A, B take the values 0, 1, 2, 3, 4) and $|^{(5)}g_0| = \psi_0^8 e^{6N_0}$ is a constant of dimensionalization determined by $|^{(5)}g|$ evaluated at $\psi = \psi_0$ and $N = N_0$. Furthermore, $^{(5)}R$ is the 5D Ricci scalar and G is the gravitational constant. In this work we shall consider $N_0 = 0$, so that $^{(5)}g_0 = \psi_0^8$. Here, the index "0" denotes the values at the end of inflation (i.e., when $\ddot{b} = 0$). With the aim to describe a manifold in apparent vacuum the Lagrangian density $\mathcal{L}$ in (1) must be only kinetic in origin

$$^{(5)}\mathcal{L}(\varphi, \varphi_{,A}) = \frac{1}{2} g^{AB}\varphi_{,A}\varphi_{,B}, \qquad (2)$$