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On the SO(9) structure of supersymmetric Yang–Mills quantum mechanics

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Abstract

In ten space–time dimensions the number of Majorana–Weyl fermions is not conserved, not only during the time evolution, but also by rotations. As a consequence the empty Fock state is not rotationally symmetric. We construct explicitly the simplest singlet state which provides the starting point for building up invariant SO(9) subspaces. The state has non-zero fermion number and is a complicated combination of the 1360 elementary, gauge invariant, gluinoless Fock states with twelve fermions. Fermionic structure of higher irreps of SO(9) is also briefly outlined.

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1. Introduction

Supersymmetric Yang–Mills quantum mechanics (SYMQM) emerge from the dimensional reduction of corresponding field theories to a single point in the $(D-1)$-dimensional space [1,2]. Resulting systems are characterized by two parameters: D—the dimensionality of the space–time of the unreduced theory, and N—the number of colors specifying a gauge group. Dependence on the gauge coupling follows from the simple rescaling of a finite number of degrees of freedom. The whole family (for various D and N) reveals a broad range of very interesting phenomena and has many applications in seemingly distant areas of theoretical physics [3–14]. Perhaps the most known example is the conjectured relevance of the $D = 10$ system, at large N, to the M-theory [15–17].

A series of results has been recently obtained for the $D = 2$ and $D = 4$, SU(2), models with the aid of the cut Fock space approach [18–20]. In this Letter we address the $D = 10$, $N = 2$ system [21] and construct explicitly the SO(9) singlet state which replaces the empty Fock state sufficient in lower dimensions. This

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state is highly non-trivial due to the non-conservation of the Majorana–Weyl fermion number in ten dimensions.

The Hamiltonian reads [1,21]

$$\begin{aligned} H &= H_K + H_P + H_F, \\ H_K &= \frac{1}{2} p_a^i p_a^i, \\ H_P &= \frac{g^2}{4} \epsilon_{abc} \epsilon_{ade} x_b^i x_c^j x_d^i x_e^j, \\ H_F &= \frac{ig}{2} \epsilon_{abc} \psi_a^\dagger \Gamma^k \psi_b x_c^k. \end{aligned} \tag{1}$$

There are 27 bosonic coordinates x_a^i, and their momenta p_a^i, $i = 1, \dots, 9$, $a = 1, 2, 3$. Fermionic degrees of freedom compose a Majorana–Weyl spinor in the adjoint representation of the SU(2) gauge group, ψ_a^α, $\alpha = 1, \dots, 16$. Γ^k are 16×16 subblocks of the big (32×32) Dirac α^k matrices in chiral representation. In all explicit calculations we use the representation of Ref. [22].

The system has the internal Spin(9) rotational symmetry generated by the gauge invariant angular momentum

$$J^{kl} = \left(x_a^{[k} p_a^{l]} + \frac{1}{2} \psi_a^\dagger \Sigma^{kl} \psi_a \right), \tag{2}$$

with

$$\Sigma^{kl} = -\frac{i}{4} [\Gamma^k, \Gamma^l]. \tag{3}$$

After the dimensional reduction, the local gauge invariance amounts to the global invariance under the SU(2) rotations generated by the color angular momentum

$$G_a = \epsilon_{abc} \left(x_b^k p_c^k - \frac{i}{2} \psi_b^\dagger \psi_c \right). \tag{4}$$

Furthermore, the Hamiltonian, Eq. (1), is invariant under, $\mathcal{N} = 1$, ten-dimensional supersymmetry with 16 Majorana–Weyl generators

$$Q_\alpha = (\Gamma^k \psi_a)_\alpha p_a^k + ig \epsilon_{abc} (\Sigma^{jk} \psi_a)_\alpha x_b^j x_c^k. \tag{5}$$

Supersymmetry requires imposing both Weyl and Majorana conditions on the 32-dimensional spinor. To identify explicitly fermionic degrees of freedom, we construct big (32×32) Dirac α matrices following Ref. [22]. In chiral representation they are block diagonal, hence we restrict ourselves to one chirality.[1] It is well known that both Majorana and Weyl conditions can be simultaneously imposed only in $D = 2 \pmod 8$ space–time dimensions. Consequently the Majorana matrix turns out to be block diagonal as well, and we can impose Majorana condition in one chirality subblock. Finally, a solution of the Majorana condition, in chiral representation, has a simple form

$$\begin{aligned} \psi_a^T = \big(& f^1, f^2, f^3, f^4, f^5, f^6, f^7, f^8, f^{8\dagger}, -f^{7\dagger}, \\ & f^{6\dagger}, -f^{5\dagger}, -f^{4\dagger}, f^{3\dagger}, -f^{2\dagger}, f^{1\dagger} \big)_a \end{aligned} \tag{6}$$

with $f^\dagger$ and f being the standard, anticommuting, fermionic creation and annihilation operators. Therefore, the ten-dimensional supersymmetric Yang–Mills quantum mechanics, with SU(2) gauge group has 24 fermionic degrees of freedom.

2. The cut Fock space approach

There exists surprisingly powerful method to compute the complete spectrum and eigenstates of polynomial Hamiltonians with a “reasonably” large number of degrees of freedom [18]. It was applied successfully to the $D = 2$ and $D = 4$ SYMQM with 6 and 15 degrees of freedom respectively [19,20]. In the latter case many new properties of this system were uncovered, including identification of dynamical supermultiplets, computation of their energies, wave functions, etc. Ten-dimensional system can also be attacked with this approach. Presumably the high accuracy of Ref. [20] could not be matched at the moment, but the recursive technique developed there offers a real possibility for some quantitative results. However, the ten-dimensional system is more complex, also on the more fundamental level. Namely, the fermion number is not conserved by the Hamiltonian, and also by the SO(9) rotations, Eq. (2) [1]. In this work we address this difficulty in some detail and propose one possible solution.

In order to better illustrate the problem, we briefly sketch the method of Refs. [18–20]. Since the Hamiltonian, Eq. (1) is a simple function of creation and annihilation operators, it is convenient use the eigenbasis

[1] Compared to Ref. [22] simple similarity transformation is required to bring big α matrices to this form.