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# Higgs boson mass and electroweak–gravity hierarchy from dynamical gauge–Higgs unification in the warped spacetime

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**Abstract**

Dynamical electroweak symmetry breaking by the Hosotani mechanism in the Randall–Sundrum warped spacetime is examined, relations among the W-boson mass ($m_{\rm W}$), the Kaluza–Klein mass scale ($M_{\rm KK}$), and the Higgs boson mass ($m_{\rm H}$) being derived. It is shown that $M_{\rm KK}/m_{\rm W} \sim (2\pi kR)^{1/2}(\pi/\theta_{\rm W})$ and $m_{\rm H}/m_{\rm W} \sim 0.058kR(\pi/\theta_{\rm W})$, where $k^2$, $R$, and $\theta_{\rm W}$ are the curvature and size of the extra-dimensional space and the Wilson line phase determined dynamically. For typical values $kR = 12$ and $\theta_{\rm W} = (0.2\text{–}0.4)\pi$, one finds that $M_{\rm KK} = 1.7$–$3.5$ TeV, $k = (1.3\text{–}2.6) \times 10^{19}$ GeV, and $m_{\rm H} = 140$–$280$ GeV.


Although the standard model of the electroweak interactions has been successful to account for all the experimental data so far observed, there remain a few major issues to be settled. First of all, Higgs particles are yet to be discovered. The Higgs sector of the standard model is for the most part unconstrained unlike the gauge sector where the gauge principle regulates the interactions among matter. Secondly, the origin of the scale of the electroweak interactions characterized by the W-boson mass $m_{\rm W} \sim 80$ GeV or the vacuum expectation value of the Higgs field $v \sim 246$ GeV becomes mysterious once one tries to unify the electroweak interactions with the strong interactions in the framework of grand unified theory, or with gravity, where the energy scale is given by $M_{\rm GUT} \sim 10^{15}$–$10^{17}$ GeV or $M_{\rm Pl} \sim 10^{19}$ GeV, respectively. The natural explanation of such hierarchy in the energy scales is desirable. In this Letter we show that the Higgs sector of the electroweak interactions can be integrated in the gauge sector, and the electroweak energy scale is naturally placed with the gravity scale within the framework of dynamical gauge–Higgs unification in the Randall–Sundrum warped spacetime.

The scheme of dynamical gauge–Higgs unification was put forward long time ago in the context of higher-dimensional non-Abelian gauge theory with non-simply connected extra-dimensional space [1,2]. In non-simply

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connected space there appear non-Abelian Aharonov–Bohm phases, or Wilson line phases, which can dynamically induce gauge symmetry breaking even within configurations of vanishing field strengths. The extra-dimensional components of gauge potentials play a role of Higgs fields in four dimensions. The Higgs fields are unified with the gauge fields and the gauge symmetry is dynamically broken at the quantum level. It was originally designed that Higgs fields in the adjoint representation in $SU(5)$ grand unified theory are unified with the gauge fields.

The attempt to identify scalar fields as parts of gauge fields was made earlier by utilizing symmetry reduction. Witten observed that gauge theory in four-dimensional Minkowski spacetime with spherical symmetry reduces to a system of gauge fields and scalar fields in two-dimensional curved spacetime [3]. This idea was extended to six-dimensional gauge theory by Fairlie [4] and by Forgacs and Manton [5] to accommodate the electroweak theory in four dimensions. It was recognized there that to yield $SU(2)_L \times U(1)_Y$ symmetry of electroweak interactions in four dimensions one need start with a larger gauge group such as $SU(3)$, $SO(5)$ or $G_2$. The reduction of the symmetry to $SU(2)_L \times U(1)_Y$ was made by an ad hoc ansatz for field configurations in the extra-dimensional space. For instance, Manton assumed spherically symmetric configurations in the extra-dimensional space $S^2$. As was pointed out later [6], such a configuration can be realized by a monopole configuration on $S^2$.[1] However, classical non-vanishing field strengths in the background would lead to the instability of the system. In this regard gauge theory defined on non-simply connected spacetime has big advantage in the sense that even with vanishing field strengths Wilson line phases become dynamical and can induce symmetry breaking at the quantum level by the Hosotani mechanism.

Recently significant progress has been achieved along this line by considering gauge theory on orbifolds which are obtained by modding out non-simply connected space by discrete symmetry such as $Z_n$ [7–21]. With the orbifold symmetry breaking induced from boundary conditions at fixed points of the orbifold, a part of light modes in the Kaluza–Klein tower expansion of fields are eliminated from the spectrum at low energies so that chiral fermions in four dimensions naturally emerge [7]. Further, in $SU(5)$ grand unified theory (GUT) on orbifolds the triplet–doublet mass splitting problem of the Higgs fields [10] and the gauge hierarchy problem [8] can be naturally solved.

The orbifold symmetry breaking, however, accompanies indeterminacy in theory. It poses the arbitrariness problem of boundary conditions [15]. One needs to show how and why a particular set of boundary conditions is chosen naturally or dynamically, which is achieved, though partially, in the scheme of dynamical gauge–Higgs unification.

Quantum dynamics of Wilson line phases in GUT on orbifolds was first examined in Ref. [14] where it was shown that the physical symmetry is determined by the matter content. Several attempts to implement dynamical gauge–Higgs unification in the electroweak theory have been made since then. The most intriguing among those is the $U(3) \times U(3)$ model of Antoniadis, Benakli and Quiros [9]. The effective potential of the Wilson line phases in this model has been recently evaluated to show that the electroweak symmetry breaking dynamically takes place with minimal addition of heavy fermions [20]. The model is restrictive enough to predict the Kaluza–Klein mass scale ($M_{\rm KK}$) and the Higgs boson mass ($m_{\rm H}$) with the W-boson mass ($m_{\rm W}$) as an input. It turned out that $M_{\rm KK} \sim 10 m_{\rm W}$ and $m_{\rm H} \sim \sqrt{\alpha_w} m_{\rm W}$, which contradicts with the observation.

We argue that this is not a feature of the specific model examined, but is a general feature of orbifold models in which extra-dimensional space is flat. Unless tuning of matter content is enforced, the relation $m_{\rm H} \sim \sqrt{\alpha_w} m_{\rm W}$ is unavoidable in flat space as shown below. To circumvent this difficulty, it is necessary to have curved extra-dimensional space.

Randall and Sundrum introduced warped spacetime with an extra-dimensional space having topology of $S^1/Z_2$ which is five-dimensional anti-de Sitter spacetime with boundaries of two flat four-dimensional branes [22]. It was argued there that the standard model of electroweak interactions is placed on one of the branes such that the electroweak scale becomes natural compared with the Planck scale chracterizing gravity. Since then many

[1] The monopole configuration for $A_M^8$ of the $SU(3)$ gauge fields on $S^2$ realizes the envisaged symmetry reduction to $SU(2) \times U(1)$ in Ref. [5].