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Closed string thermal torus from thermo field dynamics

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Abstract

In this Letter a topological interpretation for the string thermal vacuum in the thermo field dynamics (TFD) approach is given. As a consequence, the relationship between the imaginary time and TFD formalisms is achieved when both are used to study closed strings at finite temperature. The TFD approach starts by duplicating the system's degrees of freedom, defining an auxiliary (tilde) string. In order to lead the system to finite temperature a Bogoliubov transformation is implemented. We show that the effect of this transformation is to glue together the string and the tilde string to obtain a torus. The thermal vacuum appears as the boundary state for this identification. Also, from the thermal state condition, a Kubo–Martin–Schwinger condition for the torus topology is derived.

1. Introduction

Over the years string theory has been considered the best candidate for a theory that quantizes gravity. The two most pressing areas are those of black holes physics and cosmology. However, in despite of the significant progress which has been made in the last years, there is a lot of indications that to understand the non-perturbative regime of the theory (necessary to study black holes and cosmology), a more robust theoretical framework is needed. The formulation of string theory at finite temperature has given good hints that the perturbative formulation of the theory does not work at high temperature. Owing to the exponential growth of states as function of energy, there is an upper bound temperature above which the statistical partition function diverges (the Hagedorn temperature). The very existence of the Hagedorn temperature shows that the fundamental degrees of freedom of the theory could not be the ones of the perturbative string. If the specific heat at the Hagedorn temperature is finite it denotes a phase transition, and just as quark and gluons emerge as the basic ingredients of QCD at high temperature, the true degrees of freedom of the string theory may also emerge at high temperature. So, beyond phenomenological applications, string at finite temperature may also provide important pieces of ev-

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idence of the true degrees of freedom of the theory at non-perturbative regime.

It has to be mentioned that in the last two decades there have been interesting works studying string theory at finite temperature. The standard way used to lead the string theory from zero to finite temperature is the imaginary time formalism [1–3].

In general, the statistical average of an operator Q, $\langle Q\rangle$, is defined by the functional $\omega(Q) = \mathrm{Tr}(\rho Q)$:

$$\langle Q\rangle = \frac{\mathrm{Tr}[Qe^{-\beta H}]}{\omega(1)}, \tag{1}$$

where $\rho = e^{-\beta H}$ and $\omega(1)$ is the partition function. In statistical quantum field theory the trace is taken over the Fock space formed by the field operators equipped with some algebraic properties. As a consequence of the cyclic property of the trace, the Kubo–Martin–Schwinger condition follows: $\omega(A(t)B) = \omega(BA(t + i\beta))$, which is the basis of the imaginary time formalism (Matsubara formalism).

Using the Matsubara formalism, a quantum field at finite temperature relates the functional generator in $R^{1,d-1}$ to a trace when the theory is defined in $R^{d-1} \times S^1$ with the length of the compactified circle equals to β. When the Matsubara formalism is applied to a free closed string, the theory is defined on a torus, where β is related to the imaginary part of the modulus parameter τ. The real part of τ is related to a Lagrange multiplier (λ), that imposes the S^1 isometry of the closed string to the Fock space. The gauge fixing of the S^1 isometry improves the level matching condition of the physical spectrum of the closed string. So, for the closed string, $\omega(Q)$ must be redefined as:

$$\omega(Q) = \int_0^1 d\lambda\, \mathrm{Tr}\big[Qe^{-\beta H + 2\pi i\lambda P}\big], \tag{2}$$

where P is the momentum operator that generates translations in the world-sheet σ coordinate. The integral over the Lagrange multiplier guarantees that the trace is taken only over the physical states. The partition function is defined on a torus with moduli space parameters defined by: $\tau = \lambda + i\frac{\beta}{2\pi}$. A different perspective in which the torus topology can be observed is perceived by noting that the operator $e^{(-2\pi\beta H)}$ propagates a closed string through imaginary time $i2\pi\beta$ and the operator $e^{i2\pi\lambda P}$ rotates the closed string at an $2\pi\lambda$ angle. Hence the trace corresponds to a torus constructed by gluing together the ends of an open cylinder with a relative twist [4].

On the other hand, the functional $\omega(Q)$ is called a state in algebraic statistical quantum field theory and the operators form a $C^\star$ algebra [5,6]. The functional ω resembles a vector space, so that the algebra equipped with a particular functional admits a reducible representation on a Hilbert space such as a Fock space. This is the basis of an alternative formalism to study quantum field at finite temperature developed by Takahashi and Umezawa, named thermo field dynamics (TFD) [7–12]. The TFD was developed in order to handle finite temperature with a real time operator formalism [13,14]. The main idea is to interpret the statistical average as the expectation value of Q in a thermal vacuum:

$$\frac{\omega(Q)}{\omega(1)} = \langle 0(\beta)|Q|0(\beta)\rangle. \tag{3}$$

Concerning string theory, the idea of building a thermal Fock space is particularly tempting as new degrees of freedom could be identified in this Fock space at some temperature. Although the TFD approach was adopted in the past to study first quantized bosonic string [18,19], heterotic string [20,21] and string field theory [22], it was employed within a path integral formulation. The idea of using the Fock space formulation in string theory came up in Refs. [23–28], where the thermal space was used to construct bosonic thermal boundary states interpreted as D-branes at finite temperature. To further explore the algebraic characteristics of the TFD, and to upgrade it to a powerful tool to understand string theory at finite temperature, it is first necessary to set up the connections between the TFD and the imaginary time formalisms, when both are applied to strings at thermal equilibrium. In [15], the TFD was used to derive thermodynamical quantities for type IIB superstring in a pp-wave background. It was shown that the free energy, calculated from the world-sheet torus partition function in the imaginary time formalism, can be derived from the thermal expectation value in TFD. In subsequent works the $SU(1,1)$ and $SU(2)$ thermal groups were used to generalize the TFD for applications in closed string theories [16,17]. Such a gen-