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B-meson wavefunction in the Wandzura–Wilczek approximation

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Abstract

The B-meson wavefunction has been studied with the help of the vacuum-to-meson matrix element of the non-local operators in the heavy quark effective theory. In order to obtain the Wandzura–Wilczek-type B-meson wavefunction, we solve the equations which are derived from the equation of motion of the light spectator quark in the B meson by using two different assumptions. Under the condition that $\omega_0 = 2\bar{\Lambda}$, the solutions for the B-meson wavefunction in this paper agree well with the one obtained by directly taking the heavy quark limit $m_b \to \infty$. Our results show that the equation of motion of the light spectator quark in the B meson can impose a strong constraint on the B-meson wavefunctions $\Psi_\pm(\omega, z^2)$. Based on the obtained results, we claim that both its distribution amplitudes $\phi_B(\omega)$ and $\bar{\phi}_B(\omega)$ are important for calculating the B meson decays.

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1. Introduction

Along with the theoretical and experimental progresses, B physics is attracting more and more attentions. The non-perturbative light-cone (LC) wavefunction/distribution amplitude (DA) of the B meson plays an important role in making reliable predictions for exclusive B meson decays. The B meson DA has been investigated in various approaches [1–7] and is the basis of the collinear factorization [1]. Ref. [2] shows that the B meson DA is not normalizable. Such feature of the B meson DA does not cause a problem in practice [4], but it does introduce an ambiguity in defining the B meson decay constant f_B. Recently, Ref. [8] claims that it is the B-meson wavefunction that is more relevant to the B decays and in the framework of the k_T-factorization theorem [9], they proved that

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the B-meson wavefunction is renormalizable after taking into account renormalization-group evolution effects. In the k_T-factorization theorem, by taking into account the transverse momentum dependence (k_T-dependence) into the non-perturbative wavefunctions and the hard scattering, the endpoint singularity (an example can be found in Ref. [10]) coming from the collinear factorization can be cured. Theoretically, it is an important issue to study the longitudinal and transverse momentum dependence of the B wavefunction, since it provides a major source of uncertainty in the calculations of the B decays.

Ref. [11] presents an analytic solution for the B-meson wavefunction, which satisfies the constraints coming from the equations of motion and the heavy-quark symmetry [12]. They find that the "Wandzura–Wilczek-type" contribution [13] (WW approximation), which corresponds to the valence quark distribution, can be determined uniquely in analytic form in terms of the "effective mass" ($\bar{\Lambda}$) of the meson state, which is defined in the heavy quark effective theory (HQET) [14]. However, in Ref. [11], two extra constraints for the B-meson wavefunction come from the heavy quark limit, $m_b \to \infty$. Since the mass of b-quark is limited, such condition might be too strong, and we will not take such limit in our present calculation. In the following, we shall solve the two equations that are derived from the equation of motion of the light spectator quark on the basis of some physical considerations.

2. Equations under the WW approximation

In HQET [14], the wavefunctions $\tilde{\Psi}_\pm(t, z^2)$ of the B meson can be defined in terms of the vacuum-to-meson matrix element of the non-local operators:

$$\langle 0|\bar{q}(z)\Gamma h_v(0)\big|\bar{B}(p)\big\rangle = -\frac{if_B M}{2}\,\mathrm{Tr}\left[\gamma_5\Gamma\frac{1+\not{v}}{2}\times\left\{\tilde{\Psi}_+\big(t,z^2\big) - \not{z}\,\frac{\tilde{\Psi}_+(t,z^2)-\tilde{\Psi}_-(t,z^2)}{2t}\right\}\right]. \tag{1}$$

Here, $z^\mu = (0, z^-, \mathbf{z}_\perp)$, $z^2 = -\mathbf{z}_\perp^2$, $v^2 = 1$, $t = v\cdot z$, and $p^\mu = Mv^\mu$ is the 4-momentum of the B meson with mass M. $h_v(x)$ denotes the effective b-quark field. Γ is a generic Dirac matrix. The path-ordered gauge factors are implied in between the constituent fields. Note that in the above definition, the separation between the quark and the antiquark is not restricted on the LC ($z^2 = 0$).

The effective mass ($\bar{\Lambda}$) is much smaller than the B meson mass, so the light spectator quark in the B meson can be treated as on mass shell with high precision. Based on the QCD equation of motion for the light spectator quark, we can obtain a set of equations for $\tilde{\Psi}_\pm(t, z^2)$ under the WW approximation, i.e.

$$\frac{\partial\tilde{\Psi}_-(t,z^2)}{\partial t} - \frac{\tilde{\Psi}_+(t,z^2)-\tilde{\Psi}_-(t,z^2)}{t} - \frac{z^2}{t}\frac{\partial}{\partial z^2}\big[\tilde{\Psi}_+\big(t,z^2\big)-\tilde{\Psi}_-\big(t,z^2\big)\big] = 0, \tag{2}$$

and

$$\frac{\partial\tilde{\Psi}_+(t,z^2)}{\partial t} - \frac{\partial\tilde{\Psi}_-(t,z^2)}{\partial t} - \frac{\tilde{\Psi}_+(t,z^2)-\tilde{\Psi}_-(t,z^2)}{t} + 4t\frac{\partial\tilde{\Psi}_+(\omega,z^2)}{\partial z^2} = 0. \tag{3}$$

When taking the LC limit $z^2 \to 0$, the above two equations agree well with the ones in Refs. [6,7]. By doing the Fourier transformation, $\tilde{\Psi}_\pm(t,z^2) = \int d\omega\, e^{-i\omega t}\Psi_\pm(\omega,z^2)$, Eqs. (2), (3) become that

$$\omega\frac{\partial\Psi_-(\omega,z^2)}{\partial\omega} + z^2\left(\frac{\partial\Psi_+(\omega,z^2)}{\partial z^2} - \frac{\partial\Psi_-(\omega,z^2)}{\partial z^2}\right) + \Psi_+\big(\omega,z^2\big) = 0, \tag{4}$$

and

$$\left(\omega\frac{\partial}{\partial\omega}+2\right)\big[\Psi_+\big(\omega,z^2\big)-\Psi_-\big(\omega,z^2\big)\big] + 4\frac{\partial^3\Psi_+(\omega,z^2)}{\partial\omega^2\partial z^2} = 0. \tag{5}$$