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Direct derivation of the Veneziano–Yankielowicz superpotential from matrix model

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Abstract

We derive the Veneziano–Yankielowicz superpotential directly from the matrix model by fixing the measure precisely. The essential requirement here is that the effective superpotential of the matrix model corresponding to the $\mathcal{N}=4$ supersymmetric Yang–Mills theory vanishes except for the tree gauge kinetic term. Thus we clarify the reason why the matrix model reproduces the Veneziano–Yankielowicz superpotential correctly in the Dijkgraaf–Vafa theory.

1. Introduction

It has been revealed that the connection between gauge theory and a matrix model is deep and interesting. In particular, the large-N reduced model [1] is not only useful because it reduces the dynamical degrees of freedom and thus makes the large-N gauge theory tractable, but it would provide possibly a constructive definition for a gauge theory, or even string theory [2]. For $\mathcal{N}=1$ supersymmetric gauge theory, Dijkgraaf and Vafa proposed that a simple matrix model also has enough information on the F-term of the effective superpotential [3]. More precisely, in the $\mathcal{N}=1$ supersymmetric gauge theory coupled to a chiral superfield Φ in the adjoint representation with a superpotential

$$W(\Phi)=\sum_{k=0}^{n}\frac{g_k}{k+1}\Phi^{k+1}, \tag{1.1}$$

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the prepotential $\mathcal{F}(S, g_k)$ ($S = \frac{1}{64\pi^2} \operatorname{tr} W^\alpha W_\alpha$) is equivalent to the free energy $F_m(g_m, g_k)$ of a one-matrix model

$$S_m = \frac{\hat{N}}{g_m} \operatorname{Tr} W(\Phi), \tag{1.2}$$

in the large-$\hat{N}$ limit under an identification $S = g_m$.[1]

The proofs of their proposal are given in [4,5]. In particular, it is shown in [5] by using the Konishi anomaly [6] that the Schwinger–Dyson equation for $\frac{1}{64\pi^2}\langle \operatorname{tr}(\frac{W^\alpha W_\alpha}{z-\Phi})\rangle$ is exactly the same as that for the resolvent of the matrix model $\frac{g_m}{\hat{N}}\langle \operatorname{Tr}(\frac{1}{z-\hat{\Phi}})\rangle$ in the large-$\hat{N}$ limit. Because the former and the latter is given by $\partial\mathcal{F}/\partial g_k$ and $\partial F_m/\partial g_k$ respectively, we find that the prepotential $\mathcal{F}$ and the free energy of the matrix model F_m are equivalent up to a function independent of g_k's. However, as noted in [3,5,7], the matrix model produces a stronger result than the above consideration. By taking the superpotential $W = m\Phi^2/2$ and under a suitable identification between the matrix model measure and the gauge theory cutoff, F_m can also reproduce the g_k-independent part of $\mathcal{F}$ that corresponds to the Veneziano–Yankielowicz (VY) superpotential [8]

$$S\left[\log\left(\frac{\Lambda^{3N}}{S^N}\right) + N\right], \tag{1.3}$$

where Λ is the dimensional transmutation scale associated with the gauge dynamics. In this sense, the connection between the $\mathcal{N} = 1$ gauge theory and the matrix model seems deeper than we have expected.

In [9], it is shown that the Dijkgraaf–Vafa theory can be regarded as the large-N reduction. This enables us to construct a direct map between correlators in the gauge theory and those in the matrix model and thus to show directly that equalities hold between them. From this point of view, it must be possible to find the origin of the VY superpotential in the matrix model, because we have a direct map between the gauge theory and the matrix model including the gauge field degrees of freedom.

In this Letter, we show that the matrix model indeed has an information on the pure gauge field degrees of freedom and that it can reproduce the VY superpotential. In particular, by a matrix model consideration we can derive exactly the key identification mentioned above between the measure in the matrix model and the cutoff in the gauge theory, which is just assumed in [5]. Evidently in order to do this, it is indispensable to fix the measure in the matrix model. We do this by requiring that the free energy of the matrix model corresponding to the $\mathcal{N} = 4$ supersymmetric gauge theory must vanish except for a term that corresponds to the tree gauge kinetic term. It is quite natural to fix the g_k-independent part of the free energy in this way, because it is well known that the $\mathcal{N} = 4$ gauge theory is a finite theory and does not have any quantum corrections to the holomorphic part of the effective Lagrangian [10]. Then we clarify the reason why the matrix model also reproduces the pure gauge contribution to the prepotential from the point of view of the large-N reduction.[2]

In Section 2 we determine the measure in the matrix model based on the above idea. Using this measure, we derive the VY superpotential in Section 3. Section 4 is devoted to conclusions. In Appendix A we present the derivation of the VY superpotential in the case of the broken gauge symmetry as an application of our approach.

2. Determination of the measure in the matrix model

In this section we determine the measure in the matrix model according to our requirement mentioned in the introduction.

[1] Here we have assumed that there is no gauge symmetry breaking.

[2] Derivations of the VY superpotential from the field theory point of view in the context of the Dijkgraaf–Vafa theory are given, for example, in [11,12]. In the former, it is derived by introducing fundamental matters to the $\mathcal{N} = 1$ gauge theory, while in the latter it is done by invoking the $\mathcal{N} = 4$ theory.