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# Vertex functions and infrared fixed point in Landau gauge $SU(N)$ Yang–Mills theory

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**Abstract**

The infrared behaviour of vertex functions in an $SU(N)$ Yang–Mills theory in Landau gauge is investigated employing a skeleton expansion of the Dyson–Schwinger equations. The three- and four-gluon vertices become singular if and only if all external momenta vanish while the dressing of the ghost-gluon vertex remains finite in this limit. The running coupling as extracted from either of these vertex functions possesses an infrared fixed point. In general, diagrams including ghost-loops dominate in the infrared over purely gluonic ones.


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Fifty years after the formulation of Yang–Mills theory its infrared (IR) structure is still largely unknown despite the fact that this knowledge is central to any effort in understanding the strong interactions from first principles. It has long been conjectured that IR enhancements are present. Indeed they are necessary to explain confinement. Such IR enhancements may also be the reason that no explicit glue becomes visible in the low-lying hadron mass spectrum.

Lattice calculations include in principle all nonperturbative features of Yang–Mills theories but are in practice limited by the finite lattice volume in the study of possible IR singularities [1–4]. A complementary nonperturbative continuum method is provided by the Dyson–Schwinger equations (DSEs). In Landau gauge the DSEs for the ghost and gluon propagators have been analytically solved in the IR assuming ghost dominance [5–11]. Lattice and DSEs are

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complementary and yet they agree on the propagators' IR behaviour: there is clear evidence for an IR-finite or even vanishing gluon propagator and a strongly diverging ghost propagator, in accordance with both, the Kugo–Ojima confinement criterion [12] and the Gribov–Zwanziger scenario [13].

The running of the gauge coupling is intimately related to the momentum dependence of the primitively divergent vertex functions in an $SU(N_c)$ Yang–Mills theory. For sufficiently large momenta the coupling can be calculated from perturbative corrections to these vertex functions [14]. The breakdown of perturbation theory is signaled by Landau poles, unphysical singularities at nonvanishing space-like momenta. By simply imposing analyticity for space-like momenta extrapolations to the IR have been performed in so-called analytic perturbation theory [15–17]. Typically these studies find a well-behaved coupling at all momenta and an IR fixed point. Qualitatively this agrees with the findings from the genuinely nonperturbative continuum approaches. In the latter the IR behavior of the ghost-gluon interaction in Landau gauge has been determined either from DSEs or the Exact Renormalization Group Equations (ERGEs) [8,9,18,19] and yield an IR fixed point with $\alpha(0) \approx 8.9/N_c$. The corresponding couplings from the three- and four-gluon vertex functions have not yet been studied with these techniques. Within perturbation theory the principle of gauge invariance leads to the universality of the gauge coupling: all three couplings are equal up to the well-known renormalization scheme dependencies [14,20,21] at every finite order. Although a general proof is lacking one expects such relations to also hold in the full nonperturbative theory.

In this Letter we propose a method to investigate the IR behaviour of Greens functions with an arbitrary number of ghosts and gluons. We detail the results for the gluon self-interaction vertices thus completing our knowledge about the primitively divergent vertex functions (i.e., those appearing in the renormalized Lagrangian) and the corresponding running coupling(s) in the deep IR. We construct a skeleton expansion for each DSE employing the fully dressed primitively divergent $n$-point functions of Yang–Mills theory. To all orders in this expansion the three- and four-gluon vertex functions are IR-singular if and only if all external momenta vanish. The corresponding IR exponents are hereby proportional to the IR exponent of the ghost renormalization function such that the corresponding couplings have IR fixed points. The results presented in this Letter verify the self-consistency of ghost dominance in a quite general way and prove that ghost dominance is a successful guiding principle when determining the IR behaviour of Yang–Mills Green functions in the confining phase.

In Landau gauge, the ghost and gluon propagators in Euclidean momentum space are described by the renormalization functions $G(p^2)$ and $Z(p^2)$:

$$D^G(p^2) = -\frac{G(p^2)}{p^2},$$
$$D_{\mu\nu}(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}. \tag{1}$$

On the other hand, the three- and four-point functions feature tensor structures not present in the bare Lagrangian thus requiring multiple such scalar functions. We will not specify these yet and discuss first the behaviour of the scalar amplitude multiplying only the tensor defined on the level of the bare Lagrangian. E.g., for the three-gluon vertex this tensor reads

$$\delta_{\mu_1\mu_2}(p_1-p_2)_{\mu_3} + \delta_{\mu_2\mu_3}(p_2-p_3)_{\mu_1} + \delta_{\mu_3\mu_1}(p_3-p_1)_{\mu_2}. \tag{2}$$

In a setting where all external momenta $(p_i)^2$ vanish as a single momentum scale $p^2 \to 0$ this tensor will be multiplied by an amplitude $H_1^{3g}(p^2)$ in the fully dressed three-gluon vertex. For the four-gluon and ghost-gluon vertex functions the amplitudes $H_1^{4g}(p^2)$ and $H_1^{gh\text{-}g}(p^2)$ are defined analogously. Starting from the established result [8,9,18,19]

$$Z(p^2) \to (p^2)^{2\kappa}, \qquad G(p^2) \to (p^2)^{-\kappa}, \tag{3}$$

we will show in the following that for $p^2 \to 0$

$$H_1^{3g}(p^2) \to (p^2)^{-3\kappa}, \qquad H_1^{4g}(p^2) \to (p^2)^{-4\kappa},$$
$$H_1^{gh\text{-}g}(p^2) \to \text{const}, \tag{4}$$

where the parameter $\kappa$ is determined from the propagator DSEs, see below. Eq. (3) imply that the minimum of a gluon's dispersion relation does not occur at zero momentum.[1] In addition, it concurs with

[1] The phenomenological consequences of an IR-suppressed gluon propagator can, for example, be inferred from Ref. [22].