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Infrared behaviour of massless QED in space–time dimensions $2 < d < 4$

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Abstract

We show that the logarithmic infrared divergences in electron self-energy and vertex function of massless QED in $2+1$ dimensions can be removed at all orders of $1/N$ by an appropriate choice of a non-local gauge. Thus the infrared behaviour given by the leading order in $1/N$ is not modified by higher order corrections. Our analysis gives a computational scheme for the Amati–Testa model, resulting in a non-trivial conformal invariant field theory for all space–time dimensions $2 < d < 4$.

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Massless QED in $2+1$ dimensions is of interest for various reasons. It provides a theoretical laboratory for studying the infrared (iR) divergences of perturbation theory [1–9] and chiral symmetry breaking [6–12]. It also arises naturally in several theories of high temperature superconductivity [13–15]. Also remarkably the theory is not simply super-renormalizable, it is ultraviolet (uV) finite. For a Green function with F (B) number of external fermion (boson) lines, the superficial degree of divergence is $\delta(F,B) = 4 - (3/2)F - B - L$, where L is the number of loops. Consider the possible uV divergent diagrams:

(1) One-loop fermion self-energy $\Sigma(p)$ has $\delta = 0$. But the mass renormalization is absent as a consequence of chiral symmetry. Therefore, the contribution is uV finite.

(2) One-loop vacuum polarization $\Pi_{\mu\nu}(q)$ has $\delta = 1$. But gauge invariance requires that it has the form

$$\Pi_{\mu\nu}(q) = \left(q^2\delta_{\mu\nu} - q_\mu q_\nu\right)\Pi\left(q^2\right). \tag{1}$$

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(We consider Euclidean Green functions throughout this Letter.) As two powers of the photon momenta are pulled out, $\Pi(q^2)$ effectively has $\delta = -1$ and therefore it is uV finite.

(3) Two-loop vacuum polarization has $\delta = 0$. It is also uV finite due to gauge invariance.

The same power counting shows that the iR divergences become increasingly worse with the number of loops. The iR superficial degree of divergence is given by $\Delta = -\delta$. In fact, there is a more severe type of iR divergence in perturbation theory. Self-energy insertions on any internal line of a loop give rise to infrared divergent contributions even for hard external momenta [16]. For the example shown in Fig. 1, let us perform the d^3l integration first. Clearly, the integrand will contain more and more factors of $1/l^2$ with increasing number of self-energy insertions on the photon line. There is a similar problem with self-energy insertions on any internal fermion line of a loop. As a result, perturbation theory does not exist.

A resummation of perturbation theory using $1/N$ expansion dramatically alters the situation [2]. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \sum_i \bar{\psi}^i (i\partial\!\!\!/ - eA\!\!\!/)\psi^i, \quad (2)$$

where ψ^i $(i = 1, \ldots, N)$ are N species of charged four-component spinors, all massless. Enforcing $U(2N)$ invariance [6] ensures that fermion mass is not generated to any order in perturbation theory. The charge e has an engineering dimension $1/2$. We take the large N limit with Ne^2 fixed. Then to the leading order in $1/N$, the one-loop vacuum polarization $\Pi^{(1)}(q^2)$ has to be included with the free photon propagator. Due to the masslessness of the fermion, $\Pi^{(1)}(q^2)$ is singular at $q = 0$ [1,2]:

$$\Pi^{(1)}(q^2) = \frac{\mu}{q}, \quad \mu = \frac{Ne^2}{8}. \quad (3)$$

With the conventional gauge choice,

$$D_{\mu\nu}(q) = \frac{\delta_{\mu\nu} - q_\mu q_\nu/q^2}{q^2 + \mu q} - (\alpha - 1)\frac{q_\mu q_\nu}{q^4}. \quad (4)$$

This changes the infrared behaviour of the photon propagator from being inversely quadratic to inversely linear in momentum.

We consider a rearranged perturbation theory, with this as the “free” photon propagator, but otherwise the usual fermion propagator and vertex. The only difference with the usual perturbation theory is that the *one-loop* vacuum polarization contributions are not to be included in the new diagrams. Now the ultraviolet divergences are absent in any order as before, as the free photon propagator is as usual inversely quadratic for large momenta. On the other hand, the infrared behaviour is now very different. The iR superficial degree of divergence is now

$$\Delta(F, B) = B + F - 3. \quad (5)$$

It is to be observed that this is independent of the number of loops and depends only on the number of external lines. This is analogous to the uV degree of divergence of a renormalizable theory. For non-exceptional Euclidean momenta $\{q\}$ which go to zero uniformly like

$$q = \rho Q, \quad (6)$$

the Green function is singular as $\rho^{-\Delta}$. In effect the scale dimension of photon has changed from the canonical $1/2$ to 1, while that of the fermion remains at the canonical value 1. This infrared limit corresponds to an infrared stable fixed point [6].

Subintegrations can spoil the elegant picture of the iR behaviour described above [1–4]. The danger is from subdiagrams with $\Delta = 0$ which can generate a logarithmic singularity in momenta external to this subdiagram. Thus powers of logarithms arise from various subdiagrams. These logs can shift the infrared behaviour away from that given by the naive fixed point described earlier.

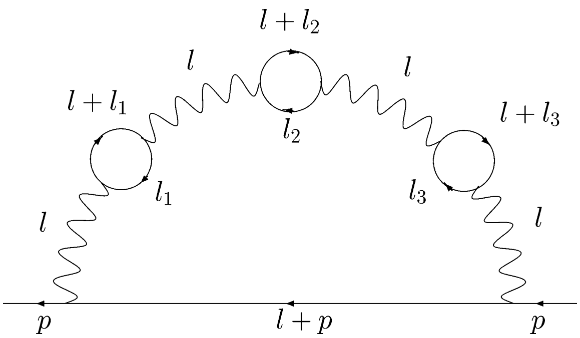

Fig. 1. If the d^3l integration is performed first, the Feynman integral diverges even for hard external momentum.