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Understanding domain wall network evolution

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Abstract

We study the cosmological evolution of domain wall networks in two and three spatial dimensions in the radiation and matter eras using a large number of high-resolution field theory simulations with a large dynamical range. We investigate the dependence of the uncertainty in key parameters characterising the evolution of the network on the size, dynamical range and number of spatial dimensions of the simulations and show that the analytic prediction compares well with the simulation results. We find that there is ample evidence from the simulations of a slow approach of domain wall networks towards a linear scaling solution. However, while at early times the uncertainty in the value of the scaling exponent is small enough for deviations from the scaling solution to be measured, at late times the error bars are much larger and no strong deviations from the scaling solution are found.

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1. Introduction

Topological defects are generic in nature and may be formed whenever a phase transition occurs. In an expanding universe cooling down from a very hot initial state it is to be expected that topological defects may provide a unique window onto the physics of the early universe offering perhaps the best hope of a clear observable link between cosmology and particle physics [1,2]. Most cosmological studies of topological defects have focused on cosmic strings due to their interesting properties and strong motivation from fundamental physics (see, for example, [3–5] and references therein). Although standard cosmic strings are now ruled out as the sole contribution to the large scale structure of the universe [6,7] they may still be the dominant source of perturbations on small cosmologi-

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cal scales and may give rise to a number of interesting cosmological consequences [8–10]. However, at present there are only two observations for which cosmic strings seem to offer the most natural explanation [11,12].

Domain wall scenarios have attracted less attention since heavy domain walls in a linear scaling regime rapidly dominate the energy density of the universe. Moreover, domain walls which are light enough to satisfy current CMB constraints have a negligible direct contribution to structure formation. However, in this case a number of interesting consequences are possible such as a contribution to the dark energy [13, 14] (if domain walls are frozen in comoving coordinates) and a small but measurable contribution to the CMB anisotropies at large angular scales [15]. Domain walls may also separate regions in the universe with different values of the cosmological parameters and/or fundamental constants of nature [16,17].

In this Letter we perform a quantitative study of the cosmological evolution of domain wall networks. We investigate the dependence of the uncertainty in key parameters characterising the evolution of the network on the size, dynamical range and number of spatial dimensions of the simulations using a simple analytic model. We then compare our analytic predictions with the results of a large set of high-resolution simulations of domain walls in two and three spatial dimensions [18], using the standard Press–Ryden–Spergel (PRS) algorithm [19] (see also [20–23]), and discuss the evidence from the simulations of a slow approach towards a linear scaling regime. Previous studies of domain wall network evolution [19–23] having a smaller number of simulations with smaller size and dynamical range than the present one have found some hints for deviations from a scale-invariant evolution. It is therefore crucial to investigate if these are only transient or if there is a more fundamental reason for such deviations.

The present Letter is a follow-up of [18]. There, we concentrated on the overall (global) dynamical features of the simulations. On the other hand, having a large dynamic range means that a more localised analysis is also possible, and in particular local exponents can be calculated with relatively small errors. In the present Letter we explore this possiblity, and also make use of the large number of simulations to discuss some analytic ways to estimate statistical errors.

2. Domain wall network evolution

We study the evolution of a domain wall network in a flat homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universe. We consider a scalar field ϕ with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} - V(\phi), \tag{1}$$

and we will take $V(\phi)$ to be the generic ϕ^4 potential with two degenerate minima given by

$$V(\phi) = V_0\left(\frac{\phi^2}{\phi_0^2} - 1\right)^2, \tag{2}$$

which obviously admits domain wall solutions. Following the procedure described in Ref. [19] we modified the equations of motion in such a way that the co-moving thickness of the domain walls is fixed in co-moving coordinates allowing us to resolve the domain walls throughout the full dynamical range of the simulations. With this modification implemented the equations of motion for the field ϕ become:

$$\frac{\partial^2\phi}{\partial\eta^2} + \alpha\left(\frac{d\ln a}{d\ln\eta}\right)\frac{\partial\phi}{\partial\eta} - \nabla^2\phi = -a^\beta\frac{\partial V}{\partial\phi}, \tag{3}$$

where a is the scale factor, η is the conformal time and α and β are constants. We take $\beta = 0$ in order to have constant co-moving thickness and $\alpha = 3$ to ensure that the momentum conservation law of the wall evolution in an expanding universe is maintained [19]. Eq. (3) is then integrated using a standard finite-difference scheme.

We have verified that the PRS algorithm gives the correct results in some special cases such as the dynamics of a plane wall or the collapse of a spherical or cilindrical domain wall. We have also verified that it appears to have a small impact on the large-scale dynamics of domain wall networks and does not seem to affect the quantities we want to measure for the purpose of testing scaling properties provided a minimum acceptable tickness is used. However, it is only possible to test the performance of the PRS algorithm over a narrow window since the 'true' equation of motions for the domain walls rapidly make the wall thickness smaller than the grid size.

In addition to these simple tests with domain walls, the PRS algorithm has been much more extensively used and tested in the case of cosmic strings (see, for