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Holographic principle and dark energy

Yun Soo Myung

Relativity Research Center and School of Computer Aided Science, Inje University, Gimhae 621-749, South Korea

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Abstract

We discuss the relationship between holographic entropy bounds and gravitating systems. In order to obtain a holographic energy density, we introduce the Bekenstein–Hawking entropy S_{BH} and its corresponding energy E_{BH} using the Friedman equation. We show that the holographic energy bound proposed by Cohen et al. comes from the Bekenstein–Hawking bound for a weakly gravitating system. Also we find that the holographic energy density with the future event horizon deriving an accelerating universe could be given by vacuum fluctuations of the energy density.

1. Introduction

Supernova (SN Ia) observations suggest that our universe is accelerating and the dark energy contributes $\Omega_{\mathrm{DE}} \simeq 0.60$–$0.70$ to the critical density of the present universe [1]. Also cosmic microwave background (CMB) observations [2] imply that the standard cosmology is given by the inflation and FRW universe [3]. A typical candidate for the dark energy is the cosmological constant. Recently Cohen et al. showed that in quantum field theory, a short distance cutoff (UV cutoff: Λ) is related to a long distance cutoff (IR cutoff: L_Λ) due to the limit set by forming a black hole [4]. In other words, if ρ_Λ is the quantum zero-point energy density caused by a UV cutoff Λ, the total energy of the system with size L_Λ should not exceed the mass of the same size-black hole: $L_\Lambda^3 \rho_\Lambda \leqslant L_\Lambda M_p^2$ with the Planck mass of $M_p^2 = 1/G$. The largest L_Λ is chosen as the one saturating this inequality and its holographic energy density is then given by $\rho_\Lambda = 3c^2 M_p^2/8\pi L_\Lambda^2$ with a numerical factor $3c^2$. Taking L_Λ as the size of the present universe, the resulting energy is comparable to the present dark energy [5]. Even though this holographic approach leads to the data, this description is incomplete because it fails to explain the dark energy-dominated present universe [6]. In order to resolve this situation, one is forced to introduce another candidates for IR cutoff. One is the particle horizon R_{h} which was used in the holographic description of cosmology by Fischler and Susskind [8]. This gives $\rho_\Lambda \sim a^{-2(1+1/c)}$ which implies $\omega_{\mathrm{h}} > -1/3$ [9]. This corresponds to a decelerating universe and unfortu-

E-mail address: ysmyung@physics.inje.ac.kr (Y.S. Myung).

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nately is not our case. In order to find an accelerating universe, we need the future event horizon R_h. With $L_\Lambda = R_h$ one finds $\rho_\Lambda \sim a^{-2(1-1/c)}$ to describe the dark energy with $\omega_h < -1/3$. This is close enough to -1 to agree with the data [1]. However, this relation seems to be rather ad hoc chosen and one has to justify whether or not $\rho_\Lambda = 3c^2 M_p^2/8\pi L_\Lambda^2$ is appropriate to describe the present universe.

On the other hand, the implications of the cosmic holographic principle have been investigated in the literature [8,10–13]. However, these focused on the decelerating universe, especially for a radiation-dominated universe.

In this Letter we will clarify how the cosmic holographic principle could be used for obtaining the holographic energy density. This together with the future event horizon is a candidate for the dark energy to derive an accelerating universe. Further we wish to seek the origin of the holographic energy density.

2. Cosmic holographic bounds

We briefly review the cosmic holographic bounds for our purpose. Let us start an $(n+1)$-dimensional Friedman–Robertson–Walker (FRW) metric

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega_{n-1}^2\right], \tag{1}$$

where a is the scale factor of the universe and $d\Omega_{n-1}^2$ denotes the line element of an $(n-1)$-dimensional unit sphere. Here $k = -1, 0, 1$ represent that the universe is open, flat, closed, respectively. A cosmological evolution is determined by the two Friedman equations

$$H^2 = \frac{16\pi G_{n+1}}{n(n-1)}\frac{E}{V} - \frac{k}{a^2},$$
$$\dot{H} = -\frac{8\pi G_{n+1}}{n-1}\left(\frac{E}{V} + p\right) + \frac{k}{a^2}, \tag{2}$$

where H represents the Hubble parameter with the definition $H = \dot{a}/a$ and the overdot stands for derivative with respect to the cosmic time t, E is the total energy of matter filling the universe, and p is its pressure. V is the volume of the universe, $V = a^n \Sigma_k^n$ with Σ_k^n being the volume of an n-dimensional space with a curvature constant k, and G_{n+1} is the Newton constant in $n+1$ dimensions. Here we assume the equation of state: $p = \omega\rho$, $\rho = E/V$. First of all, we introduce two entropies for the holographic description of a universe [14,15]:

$$S_{BV} = \frac{2\pi}{n}Ea, \qquad S_{BH} = (n-1)\frac{V}{4G_{n+1}a}, \tag{3}$$

where S_{BV} and S_{BH} are called the Bekenstein–Verlinde entropy and Bekenstein–Hawking entropy, respectively. Then, the first Friedman equation can be rewritten as

$$(Ha)^2 = 2\frac{S_{BV}}{S_{BH}} - k. \tag{4}$$

We define a quantity E_{BH} which corresponds to energy needed to form a universe-sized black hole by analogy with S_{BV}: $S_{BH} = (n-1)V/4G_{n+1}a \equiv 2\pi E_{BH}a/n$. Using these, for $Ha \leqslant \sqrt{2-k}$, one finds the Bekenstein–Hawking bound for a weakly self-gravitating system as

$$E \leqslant E_{BH} \leftrightarrow S_{BV} \leqslant S_{BH}, \tag{5}$$

while for $Ha \geqslant \sqrt{2-k}$, one finds the cosmic holographic bound for a strongly self-gravitating system as

$$E \geqslant E_{BH} \leftrightarrow S_{BV} \geqslant S_{BH}. \tag{6}$$

3. Holographic energy bounds

First we study how the gravitational holography goes well with a $(3+1)$-dimensional effective theory. For convenience we choose the volume of the system as $V_\Lambda = 4\pi L_\Lambda^3/3 \sim L_\Lambda^3$. For an effective quantum field theory in a box of volume V_Λ with a UV cutoff Λ,[1] its entropy scales extensively as [4]

$$S_\Lambda \sim L_\Lambda^3 \Lambda^3. \tag{7}$$

However, the Bekenstein postulated that the maximum entropy in a box of volume V_Λ behaves non-extensively, growing only as the enclosed area A_Λ of the box. We call it the gravitational holography. The Bekenstein entropy bound is satisfied in the effective theory if

$$S_\Lambda \sim L_\Lambda^3 \Lambda^3 \leqslant S_{BH} \equiv \frac{2}{3}\pi M_p^2 L_\Lambda^2 \sim M_p^2 L_\Lambda^2, \tag{8}$$

[1] Precisely, M_Λ is more suitable for an UV cutoff than Λ, but we here use the latter instead of M_Λ for convenience.