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Physics Letters B 607 (2005) 294–301

PHYSICS LETTERS B

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Time evolution in string field theory and T-duality

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Received 3 September 2004; accepted 16 December 2004

Available online 30 December 2004

Editor: L. Alvarez-Gaumé

Abstract

The time evolution operator (Schrödinger functional) of quantum field theory can be expressed in terms of first quantised particles moving on $\mathbb{S}^1/\mathbb{Z}_2$. We give a graphical derivation of this that generalises to second quantised string theory. T-duality then relates evolution through time t with evolution through $1/t$ and an interchange of string fields and backgrounds.

PACS: 11.25.Sq; 11.25.Tq

Keywords: String field theory; Sewing; Time evolution; T-duality

1. Introduction

In the mechanics of particles and fields it is natural to consider the evolution in time of arbitrary configurations. In second quantised string theory this is not so straightforward, for example in Witten's theory [1] the natural time variable is that at the mid-point of the string rather than a global time for the whole string. In this Letter we will construct the time evolution operator for second quantised strings by analogy with that for field theory. We begin by showing that when the field theory Schrödinger functional is written in terms of propagators expressed in first quantised form then these describe particles moving on a timelike orbifold $\mathbb{S}^1/\mathbb{Z}_2$. The first quantised propagators have an immediate generalisation to string theory, suggesting that the Schrödinger functional for second quantised strings can be expressed in terms of first quantised strings moving on this orbifold. To strengthen the analogy we give a graphical construction of the field theory Schrödinger functional which extends to both open and closed string theory. This avoids using a Lagrangian formulation of string field theory. Finally we study the effect of T-duality on time evolution and describe the nature of BRST invariance in our approach.

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2004.12.066

2. Time evolution in QFT

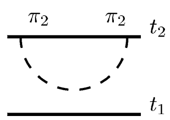
Consider a bosonic scalar field ϕ in $D+1$ dimensions. It will be convenient to work in a basis in which $\hat{\pi}(\mathbf{x}) = \dot{\phi}(\mathbf{x})$, the momentum canonically conjugate to the field,[1] is diagonal

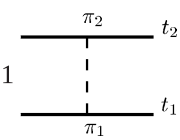
$$\langle\pi|\hat{\pi}(\mathbf{x}) = \pi(\mathbf{x})\langle\pi|, \qquad i\frac{\delta}{\delta\pi(x)}\langle\pi| = \langle\pi|\hat{\phi}(\mathbf{x}).$$

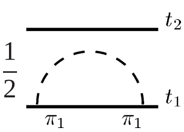
Symanzik has shown how to express the Schrödinger functional in the representation in which the field is diagonal as a functional integral [2] using sources. Generalising this to the momentum representation gives the Schrödinger functional as

$$\mathcal{S}[\pi_2, t_2; \pi_1, t_1] = \langle\pi_2|e^{-i\hat{H}(t_2-t_1)}|\pi_1\rangle = \int \mathcal{D}\varphi\, e^{iS[\varphi]+i\int\pi_2\varphi(t_2)-i\int\pi_1\varphi(t_1)}\Big|_{\dot{\varphi}(t_1)=0}^{\dot{\varphi}(t_2)=0}.$$

This has a Feynman diagram expansion in propagators which obey Neumann boundary conditions on the boundaries at times t_1 and t_2, where all external legs must end, and vertices integrated over the interval. The free field contribution is

$$\mathcal{S}[\pi_2, t_2; \pi_1, t_1] \propto \exp\Bigg(-\frac{1}{2}$$


$$+1$$


$$-\frac{1}{2}$$


$$\Bigg), \tag{1}$$

where the broken line represents the propagator, which we call G_{orb}, and the heavy lines are the spacelike boundaries. We discuss the normalisation coming from the Gaussian integral below. Without loss of generality, we will take $t_1 = 0$ and $t_2 = t$ from here on. The required boundary conditions on the propagator can be achieved using the method of images,

$$G_{\mathrm{orb}}(\mathbf{x}, t_f; \mathbf{y}, t_i) = \sum_{n\in\mathbb{Z}} G_0(\mathbf{x}, t_f + 2nt; \mathbf{y}, t_i) + \sum_{n\in\mathbb{Z}} G_0(\mathbf{x}, -t_f + 2nt; \mathbf{y}, t_i), \tag{2}$$

for $0 \leqslant t_i, t_f \leqslant t$ and G_0 is the free space propagator. To interpret this in terms of first quantisation recall that G_0 is given by a sum over paths $x(\xi)$ with an action involving an intrinsic metric g, [3]. Integrating out g gives a Boltzmann weight equal to the exponential of the length of the path,

$$G_0(x_f; x_i) = \int \mathcal{D}(x,\ g)\, e^{i\int_0^1 d\xi\,(\dot{x}\cdot\dot{x}/(2g)+m^2g/2)}\Big|_{x(0)=x_i}^{x(1)=x_f} = \int \mathcal{D}x\, e^{im\int_0^1 d\xi\,\sqrt{\dot{x}\cdot\dot{x}}}\Big|_{x(0)=x_i}^{x(1)=x_f}. \tag{3}$$

To obtain G_{orb}, we identify free space points with their images under an $\mathbb{S}^1/\mathbb{Z}_2$ (orbifold) compactification of the time direction, with radius t/π. The sum over paths to each image gives a free propagator in the sum (2).

The first form of the functional integral in (3) is immediately generalised to string theory suggesting that the Schrödinger functional for second quantised string theory can be obtained by letting the propagators in (1) represent the string propagator on the orbifold. It is not obvious how to derive this from a Lagrangian given the remarks in the introduction about the rôle of a global time in Witten's open string field theory, and given the difficulties of closed string field theory (for a review see [4] and references therein). Rather than attempt a Lagrangian derivation we will give a graphical derivation of the field theory result which can be taken over into string theory.

We appeal to a fundamental property of field theory, which follows from the observation that paths from t_3 to t_1 must cross the plane at time t_2 for $t_3 > t_2 > t_1$, so that formally the sum over paths in (3) can be factorised,

$$\sum_{\text{paths AB}} e^{-\text{length(AB)}} = \sum_{\text{C}}\Bigg(\sum_{\text{paths AC}} e^{-\text{length(AC)}}\Bigg)\Bigg(\sum_{\text{paths CB}} e^{-\text{length(CB)}}\Bigg).$$

[1] The problem of defining a momentum in string field theory goes hand in hand with the definition of a global time.