



High order smooth ambiguity preferences and asset prices[☆]



Julian Thimme^{a,*}, Clemens Völkert^b

^a Goethe University Frankfurt, Theodor-W.-Adorno-Platz 3, 60323 Frankfurt am Main, Germany

^b Finance Center Münster, Westfälische Wilhelms-Universität Münster, Universitätsstr. 14-16, 48143 Münster, Germany

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ABSTRACT

This paper extends the recursive smooth ambiguity decision model developed in Klibanoff, Marinacci, and Mukerji (2005, 2009) by relaxing the uniformity imposed on higher order acts. This generalization permits a separation of intertemporal substitution, risk attitude, and attitudes towards different sources of uncertainty. Our decision model is suited in situations where subjects may treat several kinds of uncertainty in different manners. We apply our preference specification to a consumption-based asset pricing model with long run risks and assess the impact of ambiguity on asset prices and predictability patterns. We find that modeling attitudes towards uncertainty through high order smooth ambiguity preferences has important implications for asset prices. Our model generates a highly volatile price-dividend ratio and predictability patterns in line with the data.

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1. Introduction

The subjective expected utility (SEU) theory of Savage (1954) models the behavior of a decision maker in the presence of *risk*. According to Knight (1921), risk refers to a situation in which information is described by one probability distribution. If there are two or more distinct probability measures which the DM deems possible, uncertainty about the true probability measure is considered irrelevant. However, the thought experiments of Ellsberg (1961) demonstrate that the SEU approach is inconsistent with reasonable decision making. Subjects usually prefer situations in which uncertainty concerning the true probability measure is low. This type of uncertainty is commonly called *ambiguity*.

Several models have been developed that account for ambiguity aversion.¹ A prominent example is the multiple priors model of Gilboa and Schmeidler (1989) and Epstein and Schneider (2003).

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* Corresponding author.

E-mail addresses: julian.thimme@hof.uni-frankfurt.de (J. Thimme), clemens.voelkert@gmail.com (C. Völkert).

¹ A prominent example is the multiple priors model of Gilboa and Schmeidler (1989) and Epstein and Schneider (2003). Epstein and Schneider (2010) and Etner, Jeleva, and Tallon (2012) review the literature.

Epstein and Schneider (2010) and Etner, Jeleva, and Tallon (2012) review the literature. Our approach is rooted in the smooth ambiguity model of Klibanoff, Marinacci, and Mukerji (2005), KMM in the following. In this model, the DM believes that several probability measures are possible and calculates a certainty equivalent for each of these. She then uses expected utility to arrive at a single value. Formally, their smooth ambiguity model has the following representation:

$$\int_{S_2} u_2 \left(u_1^{-1} \left[\int_{S_1} u_1 \circ f d\mu_1 \right] \right) d\mu_2^*(\mu_1).$$

For each measure μ_1 , the term in parentheses is the corresponding certainty equivalent, u_1 is the utility function which displays attitudes towards risk, while u_2 is the utility function which characterizes attitudes towards ambiguity. Similar to SEU, the ultimate probability measure (now μ_2^* on the space S_2 of probability measures) needs to be specified. This paper relaxes this assumption. If the DM has vague information about μ_2^* , this uncertainty may cause a loss of utility. We propose a preference model that accounts for this phenomenon by allowing for *high order ambiguity*.

To clarify our approach, we discuss a thought experiment. Consider the well-known two-color Ellsberg urn containing 10 balls each of which is either red or black. While a SEU-DM would fix one subjective distribution (such as 5 red and 5 black balls), a DM with smooth ambiguity preferences would consider all 11 possible compositions. Aggregation would work as follows: the DM would fix one subjective probability distribution on this set of candidate compositions and aggregate the

certainly equivalents corresponding to the respective distributions using expected utility on the second stage. Note the analogy between decision making with SEU and KMM. In both cases, the DM has to pin down one probability measure, either on the state space, or on the set of candidate distributions. We argue that both could be difficult for the DM due to uncertainty about these distributions. More specifically, uncertainty about the measure μ_2^* in the KMM representation should be accounted for. We allow the DM to consider several candidate measures μ_2 and to have different attitudes towards the different kinds of uncertainty.

In existing models of ambiguity, studying the implications of ambiguity aversion requires a classification of the sources of uncertainty into two categories (usually called *risk* and *ambiguity*). For example, one may consider the distribution of future consumption to be driven by a number of state variables. Is the investor ambiguous about one or possibly many of these factors and does she treat these sources of uncertainty equally? The usual procedure is to consider diffusive consumption uncertainty as risk and all other sources of uncertainty as ambiguity.² If we deviate from the principle that the investor evaluates all sources of uncertainty equally, it does not seem to be a sensible assumption that she categorizes them into exactly two classes about whose elements she has homogeneous tastes. Compared with standard smooth ambiguity preferences, our decision model allows for a more flexible specification of attitudes towards uncertainty. It differentiates between the sources of uncertainty and allows assigning each kind an individual preference parameter.

Ju and Miao (2012) investigate a dynamic version of the smooth ambiguity model in an endowment economy where the investor has to learn about a latent factor that drives consumption growth. They choose a hidden Markov regime-switching model, while Collard et al. (2012) use an AR(1) specification for the expected growth rate of consumption. Among other findings, both papers show that ambiguity aversion helps to generate a sizeable equity premium while keeping the risk aversion parameter at a low value. We follow Bansal and Yaron (2004) and use autoregressive processes to characterize the level and the volatility of consumption growth. Extensions of their long run risks (LRR) model introduce additional state variables and jump components.³ In contrast to this, we use a similar endowment process and focus on the preference specification. While Bansal and Yaron (2004) employ Epstein and Zin (1989) utility, EZ in the following, we use high order smooth ambiguity preferences.⁴ As our approach shares the “smoothness” of the KMM model, we are able to derive approximate analytic solutions for asset prices.

Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) calibrate their models to match important cash-flow and asset pricing moments. Constantinides and Ghosh (2011) and Beeler and Campbell (2012) identify several shortcomings of the LRR model. Two notable problems are the low volatility of the price-dividend ratio and the predictability of cash-flows and returns. We include these moments as moment conditions in a GMM estimation of the ambiguity attitude parameters and find that high aversion against uncertainty in trend growth rates helps solving these problems. Our model matches important unconditional asset pricing moments, including the volatility of the price-dividend ratio, and brings the predictive power of the price-dividend ratio for cash-flows and returns in line with the values found in the data. At the same time, high aversion to ambiguity in trend growth rates is economically plausible since current trend growth

rates are much harder to identify from observable data than current volatility levels.

The remainder of this paper is organized as follows. In Section 2, we introduce high order smooth ambiguity preferences and derive the pricing kernel. In Section 3, we exemplify the impact of high order smooth ambiguity preference on asset prices in a LRR model. Section 4 concludes.

2. High order smooth ambiguity preferences

In this section, we introduce high order smooth ambiguity preferences. We start with a static setting and then generalize to a recursive model of preference in the manner of Kreps and Porteus (1978) and Epstein and Zin (1989). Our approach extends the decision model of Klibanoff et al. (2005), which was put in a dynamic setting by Klibanoff, Marinacci, and Mukerji (2009) and Ju and Miao (2012). An axiomatic foundation is provided by Hayashi and Miao (2011). The relation between high order smooth ambiguity preferences and asset prices is discussed in Section 2.3.

2.1. The static setting

Let S_1 be a state space, equipped with a sigma-algebra Σ_1 . A first order act is a usual Savage act, i.e. a map $f : S_1 \rightarrow \mathcal{C}$ to a set \mathcal{C} of consequences. We assume that \mathcal{C} is a convex subset of \mathbb{R} .⁵ \mathcal{A}_1 denotes the set of all Σ_1 -measurable bounded first order acts. The DM's preferences are given by a binary relation \preceq_1 on \mathcal{A}_1 . If a DM agrees with the validity of certain axioms (see Savage (1954)), there is a utility function $u_1 : \mathcal{A}_1 \rightarrow \mathbb{R}$ such that she prefers an act f to an act g , i.e. $f \succeq_1 g$, if and only if $F_1(f) \geq F_1(g)$, where F_1 is the functional

$$F_1 : \mathcal{A}_1 \rightarrow \mathbb{R}, \\ f \mapsto \int_{S_1} u_1 \circ f \, d\mu_1^*.$$

Here, μ_1^* denotes a probability measure on the measure space (S_1, Σ_1) , which is assumed to be known to the DM. Another interpretation might be that she is uncertain about the true probability measure, but not averse against this kind of uncertainty. In this case, she simply aggregates different measures to the single measure μ_1^* .

If the DM is averse against ambiguity, her preferences do not permit such an aggregation. Let S_2 denote the set of probability measures on the measure space (S_1, Σ_1) .⁶ We equip S_2 with the vague topology and consider the corresponding Borel-sigma-algebra Σ_2 on S_2 . A second order act is a map $f : S_2 \rightarrow \mathcal{C}$ and the set of all Σ_2 -measurable bounded second order acts is denoted by \mathcal{A}_2 . We assume that the DM entertains a preference relation \preceq_2 on \mathcal{A}_2 .

KMM assume that the DM is able to pin down a single probability measure on (S_2, Σ_2) , called μ_2^* for the moment. This could e.g. be the case if the DM knew μ_2^* or is not averse against uncertainty about it. She would then aggregate all candidate probability measures on (S_2, Σ_2) to a single one. KMM show that a DM that accepts the validity of certain assumptions prefers the first order act $f \in \mathcal{A}_1$ to the act $g \in \mathcal{A}_1$, i.e. $f \succeq_1 g$, if and only if $F_2(f) \geq F_2(g)$, where F_2 denotes the functional

$$F_2 : \mathcal{A}_1 \rightarrow \mathbb{R}, \\ f \mapsto \int_{S_2} u_2 \left(u_1^{-1} \left[\int_{S_1} u_1 \circ f \, d\mu_1 \right] \right) d\mu_2^*(\mu_1),$$

and $u_2 : \mathcal{C} \rightarrow \mathbb{R}$ denotes a further utility function. Intuitively, the DM calculates the certainty equivalent for each possible measure μ_1 and

² See Collard, Mukerji, Sheppard, and Tallon (2012), Drechsler (2013), Ju and Miao (2012), and the references therein.

³ See e.g. Eraker and Shaliastovich (2008), Bollerslev, Tauchen, and Zhou (2009), Benzoni, Collin-Dufresne, and Goldstein (2011), and Drechsler and Yaron (2011).

⁴ Bonomo, Garcia, Meddahi, and Tedongap (2011) also explore the endowment process of Bansal and Yaron (2004) using “exotic” preferences. They consider the generalized disappointment aversion of Routledge and Zin (2010) and find that their model can improve upon the benchmark LRR model.

⁵ More generally, one may assume that \mathcal{C} is a connected separable topological space, see Ghirardato and Marinacci (2003).

⁶ Klibanoff et al. (2005) consider the state space $S_1 = \Omega \times (0, 1]$. They define S_2 as the set of all countably additive product probability measures with the Lebesgue measure on the Borel-sigma-algebra on $(0, 1]$. Our treatment does not require such a specification.

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