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# How much can lack of marketability affect private equity fund values?

Axel Buchner *

*Department of Business and Economics, University of Passau, 94030 Passau, Germany*

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ABSTRACT

This paper derives an upper bound on the discounts for lack of marketability of private equity funds using option-pricing theory. The upper bound is a function of the volatility of the fund returns, of the (remaining) lifetime of the fund, of two parameters governing the speed of capital drawdowns and distributions, of the volatility of the stock market returns, and of the return correlation between the fund and the stock market. The model calibration and numerical analysis deliver several novel insights about how non-marketability affects the value: (i) upper boundary discounts are increasing functions of the return volatility of the fund, of the return volatility of the stock market, and of the average time over which a dollar committed remains invested in the fund; (ii) upper boundary discounts decrease non-linearly over the finite lifetime of a fund; (iii) estimated upper boundary discounts at the start of an average private equity fund equal $35.3 relative to $100 committed, which corresponds to an annual upper boundary return premium demanded for lack of marketability of around 7%; and (iv) estimated upper boundary discounts of venture and buyout funds are around the same magnitude, though, discounts of venture capital funds are slightly higher.



## 1. Introduction

The issue how marketability affects the value of securities is of fundamental importance in finance. In general, both the theory and the empirical evidence suggest that investors attach a lower price to assets that are not frequently traded. In private equity, this effect should even be more pronounced because stakes in private equity funds cannot readily be sold and investors are locked-in a fund investment for quite long time periods, usually in the order of ten to fifteen years. This has dramatically been illustrated during the financial crisis through the failed attempts of large private equity investors (endowments such as Harvard, pension funds such as Calpers) to sell parts of their private equity portfolios at reasonable prices on the secondary private equity markets. For example, Harvard endowment has tried to sell a staggering USD1.5 billion in the year 2008 in an effort to receive cash from its private equity division, but failed to sell this stake at a reasonable price. This anecdotal evidence underlines the importance of non-marketability discounts for private equity fund investments. However, despite this importance, the magnitude and drivers of the discounts are still a largely unresolved issue in the private equity literature.

The main contribution of this paper is to derive a simple upper bound on the value of marketability of private equity funds using option-pricing theory. The paper adopts a framework that was initially proposed by Longstaff (1995) to assess the value of marketability of a stock investment and extends this framework to incorporate the main institutional features of private equity fund investments.[1] Intuitively, the upper bound developed reflects the present value of the greatest possible loss that an investor could experience by foregoing the right to sell the private equity fund at any point in time. That is, the basic idea adopted here is that non-marketability of private equity funds imposes an important opportunity cost on an investor. This can best be explained by considering a hypothetical investor that possesses perfect market timing abilities but is restricted from selling the fund over its finite legal lifetime. If the marketability restriction was relaxed, the investor could sell the fund position when it most profitable for him to do so. Then, the principle of no-arbitrage implies that the value of marketability to an investor with perfect market timing ability is simply the present value of the incremental cash flow that the investor would receive if the marketability restriction was relaxed. Clearly, this argument holds only for an investor with perfect market timing ability. If the market timing ability is imperfect, the present value of the incremental cash flow represents only an upper bound on the value of marketability. The important advantage of this framework is, however, that it does not require making all the assumptions about informational asymmetries, investor preferences, etc. that would be required in a full general equilibrium model. The framework adopted here only requires the weak assumptions of a rational investor and that the principle of

* Tel.: +49 851 509 3245; fax: +49 851 509 3242.
*E-mail addresses:* axel.buchner@uni-passau.de, axel.buchner@googlemail.com.

[1] Koziol & Sauerbier (2007) modify the approach of Longstaff (1995) to analyze possible price discounts of illiquid bonds. Chesney & Kempf (2012) also propose an approach to determine the value of tradability using option-pricing theory. However, their model differs from the Longstaff (1995) approach in the sense that the incentive to trade arises from the ability of traders to exploit a temporary pricing inefficiency in the stock market.

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no-arbitrage holds. In addition, an equilibrium model would in general also neglect that non-marketability of private equity funds imposes an important opportunity cost on investors: the foregone possibility to do market timing. It can be argued that this opportunity cost is particularly important for private equity fund investments, since private equity returns are strongly driven by cycles. This is shown by Gompers & Lerner (2000) and Kaplan & Schoar (2005) that provide supporting evidence in favor of a of boom and bust cycle in the private equity industry where capital appears to chase returns. When the private equity industry does well in general, more money flows into the industry, and many new funds enter the market. Returns then decline and the cycle begins anew.

The paper first develops a realistic model of the dynamics of private equity funds in a continuous-time setting. Following the typical construction of private equity funds, modeling fund dynamics requires three main ingredients: (i) the modeling of the commitment drawdowns; (ii) the modeling of the investment value; and (iii) the modeling of the return repayments. This model is then used to derive the no-arbitrage upper boundary value of marketability of a private equity fund using option-pricing theory. The resulting upper boundary value is a function of the return volatility of a private equity fund, of the remaining lifetime of the fund, and of two other parameters governing the speed at which capital is drawn from and distributed back to the investors over the finite fund lifetime. Further, the upper boundary value also depends on the volatility of the stock market returns and on the correlation between the private equity fund and stock market returns.

The model analysis provides a number of general theoretical insights about how non-marketability affects private equity fund values. First, it shows that upper boundary discounts for non-marketability of private equity funds decrease non-linearly over the fund lifetime. That is, a rational investor will demand the highest discount for non-marketability at the start of a fund. The non-linearity of the relationship stems from the fact that private equity fund investments involve stepwise capital drawdowns and generate intermediate capital distributions. Second, the analysis reveals that non-marketability discounts of private equity funds are increasing functions in the volatility of the fund returns and in the average time over which a dollar committed to a fund is actually invested in the fund. Third, the analysis illustrates that discounts are also increasing functions in the volatility of the stock market returns and in the correlation between the private equity fund and stock market returns. By these relationships, the model is able to explain the high discounts for stakes in private equity funds on the secondary markets that have occurred during the financial crisis. The model suggests that discounts for non-marketability of private equity funds could have risen substantially given the fact that the volatility of the fund and stock market returns was particularly high during this period. In addition, this was possibly enforced by the fact that the expected time over which a dollar committed to a fund is actually invested in the fund increased during this period due to closed exit markets, which also leads to higher discounts under the developed model setting.

In order to explore the magnitude of the upper boundary discounts for lack of marketability, the model is calibrated to cash flow data of European private equity funds that has been provided by Thomson Venture Economics (TVE). Several observations are in order. First, the results highlight that discounts for lack of marketability of private funds can potentially be sizeable. The estimated upper boundary discounts for non-marketability at the start of an average liquidated private equity fund amount to USD 35.3 relative to USD 100 committed, i.e., an investor that has committed USD 100 to a private equity fund would be willing to pay up to USD 35.3 additionally in order to obtain immediacy in liquidating the fund position. Second, estimated discounts of venture capital and buyout funds are around the same magnitude, though, upper boundary discounts of venture funds are slightly higher. This result is a direct consequence of the higher estimated return volatility of the sample venture capital funds, which increases the opportunity cost of not being able to trade. Third, by interpreting the discounts as a compensation required by investors for non-marketability, the paper derives an ex-ante upper boundary return premium required for non-marketability. This annual return premium equals around 7%. Interestingly, this figure corresponds pretty well to the risk-adjusted excess returns that recent studies document for private equity funds. For example, Ljungqvist & Richardson (2003), Harris, Jenkinson, & Kaplan (2014), and Ang, Chen, Goetzmann, & Phalippou (2013) report annual risk-adjusted excess returns between 4 and 6% for private equity funds. This may provide some indication that a large fraction of the documented outperformance of private equity funds is a compensation that investors require for non-marketability of the funds.

There are two main related literatures. The paper contributes to the scarce literature on the impact of non-marketability on the pricing and risk of private equity funds. Lerner & Schoar (2004) examine the role of transfer restrictions imposed by fund managers as a proxy of fund illiquidity and document that these restrictions are more likely in situations where asymmetric information problems are more severe. Using an index of venture capital returns and a time-series regression, Metrick (2007) estimates an annual premium of 1% for liquidity risk. Franzoni, Nowak, & Phalippou (2012) show that the returns of private equity investments are significantly related to changes in aggregate Pastor & Stambaugh (2003) market liquidity. Kleymenova, Talmor, & Vasvari (2012) investigate determinants of liquidity of private equity fund interests sold in the secondary markets. Sorensen, Wang, & Yang (2014) present a model in which they derive liquidity premia of private equity funds by solving the portfolio-choice problem of a risk-averse buy-and-hold investor. None of these studies, however, develops a model that allows assessing the possible price discounts of private equity funds for lack of marketability.

More generally, the paper is also related to other studies that evaluate the risk and return characteristics of private equity investments (e.g. Cochrane, 2005; Kaplan & Schoar, 2005; Ljungqvist, Richardson, & Wolfenzon, 2008; Phalippou & Gottschalg, 2009; Korteweg & Sorensen, 2010; Achleitner, Braun, & Engel, 2011; Robinson & Sensoy, 2011; Driessen, Lin, & Phalippou, 2012; Achleitner, Engel, & Reiner, 2013; Ang et al., 2013; Ewens, Jones, & Rhodes-Kropf, 2013; Higson & Stucke, 2013; Buchner & Stucke, 2014; Harris et al., 2014; Hochberg, Ljungqvist, & Vissing-Jorgensen, 2014). Recent empirical evidence shows that private equity investments slightly outperform traded stocks on a risk-adjusted basis. The results of this paper suggest that a premium for non-marketability could make up for a sizeable component of the risk-adjusted excess returns of private equity funds. Investors should take that into account when assessing the abnormal performance of private equity fund investments.

The rest of the paper is organized as follows. Section 2 develops the model for the cash flow and value dynamics of private funds and introduces the framework to derive the no-arbitrage upper boundary discounts for non-marketability. Section 3 calibrates the model to historical fund data and discusses the empirical implications. Finally, Section 4 concludes and identifies areas for future research. Details of the estimation methodology used are outlined in the Appendix.

## 2. The framework

The aim of this section is to develop the framework in which the upper bound on the value of marketability of private equity funds is derived. The section starts with a brief description that lays out the typical construction of private equity funds. This gives the motivation for the subsequent simple, yet realistic model for the dynamics of private equity funds. Finally, the upper bound value of marketability of a fund is derived by using option-pricing techniques.

### 2.1. Institutional setting

Investments in private equity are typically intermediated through private equity funds. Thereby, a private equity fund denotes a pooled