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Can stochastic discount factor models explain the cross-section of equity returns?☆

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ABSTRACT

We propose a multivariate test of the capital asset pricing model (C-CAPM) of the cross-sectional variation in equity returns in which we compare cross-sectional variation in equity returns to the cross-sectional variation in their conditional covariance with stochastic discount factors. We use a multivariate generalized heteroskedasticity in mean model to estimate 25 portfolios that are formed on size and the book-to-market ratio. Each portfolio is allowed to have its own no-arbitrage condition. We find that although the conditional covariances of returns with consumption exhibit negative variation across size, they do not vary across the book-to-market ratio. Thus, C-CAPM can capture the size effect, but not the value effect. The fit is, however, improved by allowing the coefficients on the consumption covariances to be different. The value effect appears to be associated with the book-to-market ratio as well as size. On its own the book-to-market ratio does not generate additional information about average returns to C-CAPM. A possible explanation for these findings is that both small and low book-to-market ratio firms are expected to have higher rates of growth.

1. Introduction

Size and value effects have long been recognized as "anomalies" both in the capital asset pricing model (CAPM) literature (summarized in Fama & French, 2006 and 2008), and in the consumption-based CAPM (C-CAPM) framework with power utility (see Cochrane, 2008). This paper tests whether stochastic discount factor (SDF) models that satisfy no-arbitrage restrictions can explain the behavior of a cross-section of returns on 25 portfolios sorted by firm size and their book-to-market ratio (the 25 Fama–French portfolios). We examine whether portfolios of stocks have different returns due to different conditional covariances between the returns and the relevant discount factors, or because the coefficients of the discount factors vary by portfolio characteristic. This provides a test of no-arbitrage as finding either effect would imply that no-arbitrage does not hold.

Instead of modeling separate no-arbitrage conditions for the returns on the 25 Fama–French portfolios, we model them simultaneously employing an SDF framework. We use a multivariate generalized autoregressive conditional heteroskasticity in mean model (MGM) as in Smith and Wickens (2002). This methodology is in contrast to most of the time-series econometric models of equity returns in the literature, which are univariate models and do not include conditional covariances (see for example Ludvigson, 2012). Smith, Sorensen, and Wickens (2008) used the approach adopted in this paper, examining various SDF models, including the standard C-CAPM, to generate models involving macroeconomic variables. Abhakorn, Smith, and Wickens (2013) estimate the MGM for the standard C-CAPM for each of the 25 Fama–French portfolios, and find that the fit of the model is significantly improved by the inclusion of the firm book-to-market value ratio (HML) factor. This paper extends their analysis by estimating all 25 Fama–French portfolio returns simultaneously and testing for each asset-pricing model whether the conditional covariances of these returns with the relevant discount factors can adequately explain the excess returns of these portfolios.

We find that C-CAPM is rejected by the no-arbitrage test. The model can explain the size effect, as the conditional covariance of consumption with firm size is negative, but not the value effect, as the conditional covariance of consumption with the book-to-market ratio does not vary as required across the book-to-market quintiles. We find that the value effect tends to be slightly lower for portfolios in the highest book-to-market quintile – indicating a lower risk premium – than for portfolios with the lowest book-to-market quintiles. Allowing the coefficients on

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the conditional covariances to vary across the portfolios improves fit markedly. As C-CAPM restricts them to be the same, this too is an indication of the failure of the model.

The paper is set out as follows. In Section 2, we briefly review the relevant literature on CAPM and C-CAPM. In Section 3 we describe our theoretical framework for asset pricing and in Section 4 we explain our econometric methodology. In Section 5, we report our empirical results. Section 6 summarizes the findings of this paper.

2. Some relevant literature on CAPM and C-CAPM

Evidence that the accounting variables firm size and the book-to-market ratio would be significant if included in the standard CAPM in addition to the market return was first presented by Fama and French (1993 and 2008). This cast doubt on the empirical validity of the CAPM as it suggested that additional pricing factors to the market return were required to successfully explain the cross-section of stock returns. This raises the question of whether such anomalies would also be significant in alternative models to CAPM such as C-CAPM which takes into account the intertemporal nature of the investor optimization problem. Cochrane (2008) found that size and value effects are not significant in C-CAPM.

More recently, however, a number of studies have attempted to explain the cross-section of equity returns using modified versions of C-CAPM that included either different or additional factors. Lettau and Ludvigson (2001) use the ratio of aggregate consumption to wealth as a conditioning variable in C-CAPM in order to better capture variations in expected returns over time. An alternative way to overcome the slowness of the consumption adjustment process was suggested by Parker and Julliard (2005) who measured the risk premium by its covariance with consumption growth cumulated over many quarters after the return period, see also Jagannathan and Wang (2007). Yogo (2006) proposed a two-factor model that includes nondurable and durable consumption growth. He found that the size and value effects are due to small and value stocks having higher durable consumption betas than large and growth stocks. Savov (2011) suggested the use of household garbage production as a proxy for consumption; as all forms of consumption produce waste, garbage growth should be informative about rates of consumption growth. These modified versions of C-CAPM seem to explain the cross-section of equity returns equally well to the Fama and French three-factor model (Fama & French, 1993). In this paper, rather than asserting that there are alternative or missing factors in C-CAPM, we exploit implications of C-CAPM that are ignored in the papers discussed above while keeping close to the ideas of Fama and French. In particular, we include the two additional factors of Fama and French, and do so using C-CAPM instead of CAPM, Thus we explore the validity of the model but in a multivariate no-arbitrage framework by estimating the 25 Fama–French portfolios simultaneously.

It appears from the results of Abhakorn et al. (2013) that in order to capture the value effect using C-CAPM it is necessary to include both firm size and the book-to-market ratio as when including them individually C-CAPM cannot explain small growth portfolios. They find that HML helps explain the 25 Fama–French portfolios across size quintiles as well as across book-to-market ratio quintiles, and suggest that HML may be associated with the investment growth prospects of firm. This could be the reason why the investment-based asset pricing models of Brennan, Wang, and Xia (2004) and Li, Vassalou, and Xing (2006) are able to explain well the cross-section of equity returns but traditional CAPM is not able to (e.g. Fama & French, 1992 and 2006 and Lewellen & Nagel, 2006). This suggests that consumption contains information about these firm characteristics that is not available through market return.

Table 1
Restrictions on the no-arbitrage condition.
s and b indicate the size and book-to-market groups for the characteristic portfolios. The numbers are in ascending order of magnitude. The smallest size is denoted by $s = 1$ while the lowest book-to-market ratio is represented by $b = 1$. γ denotes constant coefficient of relative risk aversion (CRRA). α_i represents a coefficient for each conditional covariance in Eq. (7).

Models	α_0	α_1	α_2	α_3	α_4
M1: C-CAPM with power utility and nominal return	$-\frac{1}{2}$	γ	1	0	0
M2: Restricted book-to-market C-CAPM	$-\frac{1}{2}$	$\alpha_{1,s}$	1	0	0
M3: Restricted size C-CAPM	$-\frac{1}{2}$	$\alpha_{1,b}$	1	0	0
M4: Unrestricted C-CAPM	$-\frac{1}{2}$	$\alpha_{1,sb}$	1	0	0
M5: CAPM	0	0	0	0	δ
M6: Restricted book-to-market CAPM	0	0	0	0	$\alpha_{4,s}$
M7: Restricted size CAPM	0	0	0	0	$\alpha_{4,b}$
M8: Unrestricted CAPM	0	0	0	0	$\alpha_{4,sb}$
M9: Restricted two-factor SDF model	$-\frac{1}{2}$	α_1	α_2	0	0
M10: Unrestricted two-factor SDF model	$-\frac{1}{2}$	$\alpha_{1,sb}$	$\alpha_{2,sb}$	0	0
M11: Restricted three-factor SDF model	$-\frac{1}{2}$	α_1	α_2	α_3	0
M12: Unrestricted three-factor SDF model	$-\frac{1}{2}$	$\alpha_{1,sb}$	$\alpha_{2,sb}$	$\alpha_{3,sb}$	0

3. Theoretical framework

3.1. Stochastic discount factor representations of asset pricing models

The basic no-arbitrage pricing equation for a risky asset defines a relationship between the stochastic discount factor (SDF) M_{t+1} and the risky return, R_{t+1}.

$$1 = E_t[M_{t+1}R_{t+1}] \tag{1}$$

where M_{t+1} is the stochastic discount factor for period $t+1$ and R_{t+1} is the gross nominal return on an asset (see Cochrane, 2008). If the logarithms of M_{t+1}, R_{t+1} and the risk free rate $(m_{t+1}, r_{t+1}, r_t^f)$ are jointly normally distributed, then Eq. (1) implies that the expected excess real return on equity is given by.

$$E_t\left(r_{t+1} - r_t^f\right) + \frac{1}{2}V_t(r_{t+1}) = -Cov_t(m_{t+1}, r_{t+1}) \tag{2}$$

where the term of the right-hand side is the risk premium.

Eq. (2) can also be expressed in terms of nominal returns. If i_{t+1} is the nominal return on equity, i_t^f is the nominal risk-free rate, and π_t is the inflation, the no-arbitrage condition for nominal returns is:

$$E_t\left(i_{t+1} - i_t^f\right) + \frac{1}{2}V_t(i_{t+1}) = -Cov_t(m_{t+1}, i_{t+1}) + Cov_t(\pi_{t+1}, i_{t+1}). \tag{3}$$

More generally, if m_t can be represented as a linear function of $n-1$ factors $z_{i,t}\{i = 1, \ldots, n-1\}$ so that $m_t = -\sum_{i=1}^{n-1}\alpha_i z_{i,t}$, then a general representation of Eq. (3) is

$$E_t\left(i_{t+1} - i_t^f\right) = \alpha_0 V_t(i_{t+1}) + \sum_{i=1}^{n}\alpha_i Cov_t(z_{i,t+1}, i_{t+1}), \tag{4}$$

where $z_{n,t} = \pi_t$. The differences between many asset pricing models are in their stochastic discount factor, $z_{i,t+1}$, and the restrictions imposed on the coefficients. We consider three pricing models that are special cases of Eq. (4):

(a) C-CAPM with power utility