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Annals of Physics 320 (2005) 468–486
ANNALS of PHYSICS
www.elsevier.com/locate/aop

Quantum behavior of deterministic systems with information loss: Path integral approach

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Received 26 April 2005; accepted 6 September 2005

Abstract

’t Hooft’s derivation of quantum from classical physics is analyzed by means of the classical path integral of Gozzi et al. It is shown how the key element of this procedure—the loss of information constraint—can be implemented by means of Faddeev–Jackiw’s treatment of constrained systems. It is argued that the emergent quantum systems are identical with systems obtained in Blasone et al. [Phys. Rev. A71 (2005) 052507] through Dirac–Bergmann’s analysis. We illustrate our approach with two simple examples—free particle and linear harmonic oscillator. Potential Liouville anomalies are shown to be absent.

PACS: 3.65.−w; 31.15.Kb; 45.20.Jj; 11.30.Pb

1. Introduction

The idea of quantum mechanics as the low-energy limit of some more fundamental deterministic dynamics [1,2] has been revived recently by ’t Hooft [3,4], in the attempt for a radical solution of the so-called holographic paradox, originally formulated in the context of black-hole thermodynamics [5,6].

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0003-4916/$ - see front matter
doi:10.1016/j.aop.2005.09.001

There is a widespread negative attitude towards the possibility of deriving quantum from classical physics which relies on Bell's inequalities [7]. However, although being clear that quantum mechanics at laboratory scales violates these inequalities, a common prejudice is that Bell's theorem should be true at all scales. As observed by 't Hooft [3], this need not be the case because some premisses on which the usual forms of the Bell inequalities are based may cease to hold at Planck scale.

By resorting to simple dynamical systems, 't Hooft has shown that an appropriate constraining procedure applied to the deterministic system, can reduce the physical degrees of freedom so that quantum mechanics emerges. Such a reduction of the degrees of freedom may be physically implemented by a mechanism of information loss (dissipation). This idea has been further developed by several authors [4,8–13], and it forms the basis also of this paper.

Our aim was to study 't Hooft's quantization procedure by means of path integrals, along the line of what done in our previous work [8]. However, in contrast to [8] here we treat 't Hooft's constrained dynamics by means of the Faddeev–Jackiw technique [14]. The constrained dynamics enters into 't Hooft's scheme twice: first, in the classical starting Hamiltonian which is of first order in the momenta and thus singular in the Dirac–Bergmann sense [15]. Second, in the information loss condition that one has to enforce to achieve quantization [8]. In our previous paper [8], we have adopted the customary Dirac–Bergmann technique, which is often cumbersome. Here, we want to point out the simplifications arising from the alternative Faddeev–Jackiw method, which turns out to admit a clearer exposition of the basic concepts.

The paper is organized as follows: In Section 2, we briefly discuss the main features of 't Hooft's scheme. By utilizing the Faddeev–Jackiw procedure, we present in Section 3 a Lagrangian formulation of 't Hooft's system, which allows us to quantize 't Hooft's system via path integrals in configuration space. It is shown that the fluctuating system produces a classical partition function. In Section 4, we make contact with Gozzi's superspace path integral formulation of classical mechanics. In Section 5, we introduce 't Hooft's constraint which accounts for information loss. This is again handled by means of Faddeev–Jackiw analysis. Central to this analysis is the fact that 't Hooft's condition breaks the BRST symmetry and allows to recast the classical generating functional into a form representing a genuine quantum-mechanical partition function. In Section 6, we present two simple applications of our formalism. Associated technical details of the anomaly cancellation are relegated to Appendix A. A final discussion is given in Section 7.

2. 't Hooft's quantization procedure

In this section, we briefly review the main aspects of 't Hooft's quantization procedure [4,12] to be used in this work. The basic idea is that there exists a simple class of classical systems that can be described by means of Hilbert space techniques without loosing their deterministic character. Only after enforcing certain constraints expressing information loss, one obtains bona fide quantum systems. Thus, the quantum states of actually observed degrees of freedom (*observables*) can be identified with equivalence classes of states that span the original (primordial) Hilbert space of truly existing degrees of freedom (*be-ables*).

Such a scheme is realized in certain model quantum cases where one may indeed identify the primordial systems of *be-ables* that are entirely deterministic. In discrete-time