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The radial problem in gauge field theory models

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Abstract

The study of spontaneous symmetry breaking patterns in theories in which the ground state is determined by the minima of a potential invariant under the symmetry group of the system may be traced back to the solution of two classes of problems, that we shall quote in Tolédano and Dmitriev's suggestive words [P. Tolédano, V. Dmitriev, Reconstructive Phase Transitions in Crystals and Quasicrystals, World Scientific, Singapore, 1996] as *angular* and *radial problem*, respectively. Whilst the former problem, i.e., the determination of the isotropy-type stratification, has been extensively treated both in condensed matter physics and in particle physics, the radial problem, in particular the construction of the phenomenological potential allowing the realization of all the symmetry allowed symmetry phases, has up to now substantially been disregarded in gauge field theory, because renormalizability limits to four the degree of the Higgs potential and it is widely thought that spontaneous radiative mass generation can anyway fix the issue. Through a rigorous analysis in the framework of geometric invariant theory ($\widehat{P}$-matrix approach) we review these facts, focussing our attention on the role of radiative corrections. Then, we propose a way of reconciling renormalizability requirement and tree-level observability of all the phases allowed by the symmetry. The idea will be illustrated in simple extensions of two-Higgs-doublet SM, with additional scalar singlets and discrete symmetries. This will allow us to explain the rationale behind all the extensions of the Higgs sectors so far proposed to generate the observed Baryon asymmetry of our Universe at the EW Phase Transition.

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1. Introduction

It is well known that spontaneous symmetry breaking is relevant both to elementary particle and solid-state physics, in theories in which the ground state of the system is determined by the minimum of a potential invariant under a symmetry group G. When the ground state of the system is invariant only with respect to a proper subgroup $G_0 \subset G$, the G-symmetry is said to be spontaneously broken and G_0 turns out to be the true symmetry of the system.

The classical mechanisms can be summarized in the following way. The ground state is represented by a vector ϕ_0 belonging to the Euclidean space $\mathbb{R}^n$, on which G acts (through some representation) as a group of linear transformations; ϕ_0 is determined as the point at which a G-invariant potential, the polynomial of given degree d, $V_a^{(d)}(\phi)$, assumes its absolute minimum. The potential $V_a^{(d)}(\phi)$ is the Higgs potential in a gauge field theory or the non-equilibrium free energy in a Landau theory of structural phase transitions. Generally, $V_a^{(d)}(\phi)$ is written in terms of real control parameters $a = (a_1, a_2, \ldots)$, determined by external conditions (for instance, scalar self-couplings in Higgs potentials, or pressure and temperature in the free energy), that are not subjected to symmetry bounds. As a consequence, the location of the absolute minimum ϕ_0 and the residual symmetry G_0 can depend on the a's, and various patterns of spontaneous symmetry breaking are allowed, corresponding to distinct structural phases of the system.

Owing to the degeneracy due to the symmetry, the complete analytical determination of the minima of the potential is, generally, a difficult computational task. It is now well established that the privileged framework to deal with spontaneous symmetry breaking and, generally, bifurcation in the presence of symmetry, is geometric invariant theory (see, for instance, the monographs [2–5] or [6] and references therein).

In fact, the absolute minimum of $V_a^{(d)}(\phi)$ is degenerate along a G-orbit Ω_0, whose points define equivalent ground states. The set of subgroups of G that leave invariant (*isotropy subgroups of G at*) the points of Ω_0 form a conjugacy class $[G_0] = \{gG_0g^{-1} | g \in G\}$, that defines both the *orbit type* of Ω_0 and the residual symmetry of the system after spontaneous symmetry breaking. We shall think of this symmetry as thoroughly characterizing the *phase* of the system. Distinct G-orbits can have the same symmetry and orbits with the same symmetry are said to form a *stratum*. Minima of the Higgs potential located at orbits lying in the same stratum determine the same phase: *there is a one-to-one correspondence between strata and phases allowed by the G-symmetry*.

The simple observation that, like any G-invariant function, $V_a^{(d)}(\phi)$ can be expressed as a function of a finite set $p(\phi) = (p_1(\phi), \ldots, p_q(\phi))$ of basic polynomial invariants implies that when the point $p \in \mathbb{R}^q$ ranges in the domain spanned by $p(\phi)$,