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From quantum mechanics to classical statistical physics: Generalized Rokhsar–Kivelson Hamiltonians and the "Stochastic Matrix Form" decomposition

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Abstract

Quantum Hamiltonians that are fine-tuned to their so-called Rokhsar–Kivelson (RK) points, first presented in the context of quantum dimer models, are defined by their representations in preferred bases in which their ground state wave functions are intimately related to the partition functions of combinatorial problems of classical statistical physics. We show that all the known examples of quantum Hamiltonians, when fine-tuned to their RK points, belong to a larger class of real, symmetric, and irreducible matrices that admit what we dub a Stochastic Matrix Form (SMF) decomposition. Matrices that are SMF decomposable are shown to be in one-to-one correspondence with stochastic classical systems described by a Master equation of the matrix type, hence their name. It then follows that the equilibrium partition function of the stochastic classical system partly controls the zero-temperature quantum phase diagram, while the relaxation rates of the stochastic classical system coincide with the excitation spectrum of the quantum problem. Given a generic quantum Hamiltonian construed as an abstract operator defined on some Hilbert space, we prove that there exists a continuous

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manifold of bases in which the representation of the quantum Hamiltonian is SMF decomposable, i.e., there is a (continuous) manifold of distinct stochastic classical systems related to the same quantum problem. Finally, we illustrate with three examples of Hamiltonians fine-tuned to their RK points, the triangular quantum dimer model, the quantum eight-vertex model, and the quantum three-coloring model on the honeycomb lattice, how they can be understood within our framework, and how this allows for immediate generalizations, e.g., by adding non-trivial interactions to these models.

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1. Introduction

The combinatorial problem of counting how many ways there are to pack dimers on some given lattice is of relevance to chemists, physicists, and mathematicians [1–4]. A deep connection between the statistical physics of closely packed, hard-core dimers on two-dimensional lattices, and the critical behavior of the two-dimensional Ising model was established in the early 1960s [2,3]. Soon after the discovery of high-temperature superconductivity, a quantum version of the classical hard-core dimer problem on the square lattice was proposed by Kivelson, Rokhsar, and Sethna as an effective low energy theory for a doped Mott insulator [5–11].

This example of a square lattice quantum dimer model is interesting and unusual in several ways. (1) There is a one-to-one correspondence between the (dimer) basis that spans the underlying Hilbert space and the configuration space of the combinatorial problem. (2) The quantum Hamiltonian is the sum over local Hermitian operators, each of which encodes the competition between a potential energy that favors a local ordering of dimers and a kinetic energy term that favors a local quantum superposition of dimers, i.e., a local quantum liquid state. (3) For a special value of the ratio between the characteristic potential and kinetic energies called the Rokhsar–Kivelson (RK) point, the local Hermitian operators entering the Hamiltonian are positive semidefinite and the ground state (GS) is the equal-weight superposition of all dimer states, i.e., the normalization of the GS is nothing but the number of ways to closely pack hard-core dimers on the square lattice [6]. Remarkably, this GS is a very peculiar liquid state since it is critical according to the results of Kasteleyn on the classical square lattice dimer model [2]. (4) It was realized by Henley that the excitation spectrum of the quantum square lattice dimer model at the RK point is identical to the spectrum of relaxation rates of the classical square lattice dimer model out of thermal equilibrium when equipped with a properly chosen Monte Carlo dynamics [12].

Building on the close interplay between the classical and quantum square lattice dimer models, Moessner and Sondhi extended the triangular lattice classical dimer model to a quantum one and showed that its RK point realizes an incompressible spin