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Quantum nonintegrability and the classical limit for *usp*(4) systems

Marcel Novaes* and José Eduardo M. Hornos

Instituto de Física de São Carlos—Universidade de São Paulo, CP 369, 13560-970, São Carlos, SP, Brazil

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Abstract

We investigate the transition from integrability to chaos in a system built of *usp*(4) elements, both in the quantum case and in its classical limit, obtained using coherent states. This algebraic Hamiltonian consists in an integrable term plus a nonlinear perturbation, and we see that the level spacing distribution for the quantum system is well approximated by the Berry–Robnik–Brody distribution, and accordingly the classical limit displays mixed dynamics.

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1. Introduction

The level spacing distribution $P(s)$ of a quantum system is considered to be universally related to the type of dynamics of its classical limit. Integrable systems with more than one degree of freedom display level clustering, or more precisely an exponential law for $P(s)$. Completely chaotic systems, on the other hand, are characterized by level repulsion and well described by the different Wigner surmises obtained from random-matrix theory [1,2].

How does the transition form integrability to chaos take place on the quantum level, or what should be the form of $P(s)$ for a system with mixed classical dynamics,

* Corresponding author. Present address: Instituto de Física "Gleb Wataghin", Universidade Estadual de campinas, 13083-970 Campinas-SP, Brazil. Fax: +551933885496.
E-mail address: mnovaes@ifi.unicamp.br (M. Novaes).

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is not completely understood. An important result in this direction was that of Berry and Robnik [3], who derived a semiclassical formula (valid for $\hbar \to 0$) for $P(s)$ assuming statistical independence of different phase space regions. The Berry–Robnik distribution has a parameter ρ, the fraction of the energy surface for which the motion is regular, and interpolates smoothly between the exponential law and the random-matrix distributions.

When $\hbar$ is small but finite the Berry–Robnik distribution does not necessarily provide an accurate approximation for the actual system. In this phenomenological level the Berry–Robnik–Brody [4] distribution has proved useful, although it still lacks a rigorous theoretical derivation. In this work, we study a quantum system which, on the classical level, has a mixed phase space. Its Hamiltonian, written in terms of the generators of the unitary symplectic algebra $usp(4)$, consists in an integrable term plus a nonlinear perturbation with coupling constant ϵ, and its classical limit is obtained using coherent states. We find that the classical limit has mixed type of dynamics, and show the Berry–Robnik–Brody distribution to be a good description of its level spacing distribution.

Algebraic Hamiltonians have proved useful in reproducing qualitatively and quantitatively the properties of observed spectra and transition probabilities [5,6]. The success of the Interacting Boson Model introduced by Arima and Iachello [5] in nuclear physics led to the development of similar techniques in molecular physics [6,7] and condensed matter [8]. These algebraic approaches often involve the construction of coset spaces and coherent states of Lie groups [7,8]. An algebraic approach also exists to the degeneracies of the genetic code [9], which makes use of the unitary symplectic group $USp(6)$.

Coherent states are generally considered to behave as closest-to-classical quantum states [10], and have proved to be a useful tool in obtaining the classical limit of quantum systems with Lie group symmetry [11,12]. It is well known that in some cases the manifold of coherent states is a classical phase space, and that the quantum evolution reduces to Hamilton equations on this space in the semiclassical limit $\hbar \to 0$. They are also at the heart of the converse procedure, that of quantizing a given classical system [13].

Coherent states for the unitary symplectic group $USp(4)$ were recently studied in detail in [14], where the classical phase space and canonical coordinates were obtained. These basic results will be reviewed in Section 4, after we introduce the symplectic group and its irreducible representations. In Section 5, we present a brief discussion of the level spacing distribution $P(s)$ for systems with mixed phase space. A particular Hamiltonian is defined in Section 6, and its $P(s)$ is seen to be closely related to the classical dynamics. We conclude in Section 7.

2. The symplectic group

The symplectic group is a basic symmetry group of both classical and quantum mechanics. The conjugate variables of classical mechanics are defined by their Poisson bracket relation $\{q_i, p_j\} = \delta_{ij}$ $(i, j = 1 \dots N)$, while in quantum mechanics we