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Annals of Physics 315 (2005) 505–531
ANNALS of PHYSICS
www.elsevier.com/locate/aop

Computing the wavefunction from trajectories: particle and wave pictures in quantum mechanics and their relation

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Received 25 May 2004; accepted 16 September 2004
Available online 16 December 2004

Abstract

We describe the particle method in quantum mechanics which provides an exact scheme to calculate the time-dependent wavefunction from a single-valued continuum of determinstic trajectories where two spacetime points are linked by at most a single orbit. A natural language for the theory is offered by the hydrodynamic analogy, in which wave mechanics corresponds to the Eulerian picture and the particle theory to the Lagrangian picture. The Lagrangian model for the quantum fluid may be developed from a variational principle. The Euler–Lagrange equations imply a fourth-order nonlinear partial differential equation to calculate the trajectories of the fluid particles as functions of their initial coordinates using as input the initial wavefunction. The admissible solutions are those consistent with quasi-potential flow. The effect of the superposition principle is represented via a nonclassical force on each particle. The wavefunction is computed via the standard map between the Lagrangian coordinates and the Eulerian fields, which provides the analogue in this model of Huygens' principle in wave mechanics. The method is illustrated by calculating the time-dependence of a free Gaussian wavefunction. The Eulerian and Lagrangian pictures are complementary descriptions of a quantum process in that they have associated Hamiltonian formulations that are connected by a canonical transformation. The de Broglie–Bohm interpretation, which employs the same set of trajectories, should not be conflated with the Lagrangian version of the hydrodynamic interpretation. The theory implies that the mathematical results of the de

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0003-4916/$ - see front matter
doi:10.1016/j.aop.2004.09.008

Broglie–Bohm model may be regarded as statements about quantum mechanics itself rather than about its interpretation.

PACS: 03.65.Bz
Keywords: Quantum mechanics; Hydrodynamics; Particle trajectories; Variational principle; Canonical transformation

1. Introduction

The notion that the concept of a continuous material orbit is incompatible with a full wave theory of microphysical systems was central to the genesis of wave mechanics [1]. Early attempts to justify this assertion using Heisenberg's relations were subsequently shown to be flawed, and indeed no credible proof forbidding the treatment of quantum processes in terms of precisely defined spacetime trajectories has ever been offered. The idea nevertheless entered the folklore of the subject and even now is invoked to highlight alleged paradoxical implications of quantum mechanics (e.g., Schrödinger's cat). So great was the philosophical bias that not only was the material orbit ruled out as an aid to comprehension, but the possibilities of using the trajectory as a computational tool, or even as the basis of an alternative representation of the quantum theory—the twin subjects of this paper—were foregone.

In the path-integral method the path concept is integral to the computation of the wavefunction: the propagator linking two spacetime points is calculated by linearly superposing the elementary amplitudes associated with all the paths connecting the points [2]. The wavefunction at a point is then found from Huygens' principle by superposing the contributions coming from all the other points (weighted by the initial wavefunction). The technique gives the impression that the trajectory concept works in this context as a computational device only because it is allied with a simultaneous application of the superposition principle to the path. Thus, the propagation of a particle somehow involves the simultaneous traversal of multiple equally likely paths. The method therefore rather reinforces the view that a description of the propagation between two points using a *single* spacetime track is incompatible with the quantum description of the process.

Of course, de Broglie and Bohm demonstrated long ago the falseness of this conclusion—no rule of quantum theory is broken by employing a single trajectory to specify the state of a system more finely than does the wavefunction [3,4]. But, being just an interpretative element rather than an intrinsic ingredient of quantum theory, it has been possible to dismiss the de Broglie–Bohm trajectory as a "superfluous 'ideological superstructure'" [5]. Indeed, the very method of computing the trajectory, which relies on first knowing the wavefunction whose determination is independent of the path, appears to confirm the de Broglie–Bohm trajectory as a redundant addendum.

The question whether the concept of a single path linking two spacetime points, such as that employed by de Broglie and Bohm, can be made the basis of an alternative computational scheme in quantum mechanics has received scant attention in