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Characterizing chaotic response of a squid axon through generating partitions

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Abstract

We apply a recently proposed method for estimating generating partitions [Phys. Rev. E 70 (2004) 016215] to data of the chaotic response observed in a squid giant axon. Using the estimated partitions we argue that given the data, a binary symbol sequence of “firing” and “non-firing” leads to a complete description of the dynamics.

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1. Introduction

Nerve membranes respond chaotically to periodic stimulations with appropriate amplitudes and periods under a space-clamped condition [1,2]. The chaotic response is often described as a sequence of electrical spikes, called action potentials. Neural information is believed to be carried by action potentials, although it is still an important open question how action potentials encode information, especially in higher brain areas. Since firing of a neuron usually occurs when the membrane potential exceeds a threshold in a trigger zone, one can view firing patterns as a symbol sequence obtained by thresholding the membrane potential time series. Bollt et al. [3] have warned, however, that, in general, using a threshold for generating symbolic dynamics in a high-dimensional system can give misleading results. That is, the symbol sequences obtained are not necessarily sufficient to completely describe the system’s dynamics.

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In this Letter, we present evidence that a thresholding of neuronal data, which results in a firing and non-firing sequence, is valid for the particular experimental data sets we examine. The observed data set we use records the response of a squid giant axon to periodic stimulation [2]. Our method is to estimate a generating partition from the data using a recently proposed technique [4], then verify that the firing response and the estimated generating partition give equivalent symbolic dynamics.

2. Background

For our purpose a *partition* is a set of disjoint subsets of a state space whose union covers the whole state space. Each point in the state space belongs to just one element of the partition, because the partitions are complete and disjoint. By assigning a symbol to each element of the partition, then each point in the state space is labeled with a symbol. Corresponding to a time series $\ldots, x_{i-1}, x_i, x_{i+1}, \ldots$ of states, there is a symbol sequence $\ldots, s_{i-1}, s_i, s_{i+1}, \ldots$.

Suppose that the time series is generated from a deterministic system described by an invertible map. Then knowledge of a symbol sequence s_i allows one to locate the partition element that contains the state x_i. For certain systems given knowledge of a substring $s_{i-m}s_{i-m+1}\cdots s_{i+n}$ allows one to accurately locate the state x_i by considering the intersections of images and pre-images of the partition sets [5]. For notational convenience, specify the *center* of a substring, by marking it with a "$*$", that is, $s_{i-m}s_{i-m+1}\cdots s_{i-1} * s_i \cdots s_{i+n}$, and call this a *substring* for x_i. Generally, a longer substring localizes the state x_i better.

We call a partition *generating* if any two states can be distinguished by some finite length substring, except possibly for a set of states of measure zero [6]. This means that with probability one, a state, or a trajectory, is uniquely identified using an infinite symbol sequence, which is the bi-directional extension of the substrings. For this reason there is much interest in finding generating partitions of dynamical systems [4, 7–10], because the dynamical system is (almost everywhere) equivalent to the symbolic dynamics of a generating partition, that is, the symbolic sequence generated by a generating partition is (almost) the complete description of the dynamical system. Recently some methods for estimating a generating partition have been proposed using a set of unstable periodic points [8,9] or simply from a time series [4,10].

If a partition is close to being generating, one can expect that states with the same substring are close to each other, and, therefore, they are well represented by a single point. A new method for estimating generating partitions for time series exploits this property [4]. In this method, each substring is assigned a point, called a *representative*, which roughly locates all states labeled with that substring. Then, the problem for finding a generating partition is formulated as a mixed optimization problem, which minimizes the discrepancy, or the sum of the squared errors, between states in a time series and the corresponding time series of symbols and representatives. By this method, good estimates of generating partitions were obtained for the Hénon and Ikeda maps, even from a short and noisy time series [4]. The method is referred to the *symbolic shadowing* method.

3. The data

The neuronal data set we used is shown in Fig. 1(a). These are periodically observed membrane potentials normalized to lie between 0 and 1. The time series has 499 points, which we denote by $\{u_t\}_{t=1}^{499}$. There has been an attempt at obtaining predictive models for this time series [2], the success of which suggests that this time series is well modeled as a low-dimensional chaotic system. There is some concern, however, that the time series includes a transient, or diplays intermittency; to avoid this difficulty we use, in this Letter, only the last 400 consecutive points $\{u_t\}_{t=100}^{499}$ and assume that this is typical behavior. The return plot is shown in Fig. 1(b), which can be regarded as evidence of low-dimensional dynamics, because it does not fill this two-dimensional space.

Firing and non-firing of the nerve membrane are experimentally observed, but in this data set can be identified as a state where $u_{t+1} < 0.45$, which is approximately those states where $u_t > 0.88$, however, as a consequence of steep separatrix and experimental noise, the latter is not an accurate characterization, so we use the former. Hence, in Fig. 1(b) we identify the points in region F as the firing states, and points in regions N_1 and N_2 the non-firing states.