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Dynamics of a class of cellular neural networks with time-varying delays ☆

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Abstract

Employing Brouwer's fixed point theorem, matrix theory, a continuation theorem of the coincidence degree and inequality analysis, the authors make a further investigation of a class of cellular neural networks with delays (DCNNs) in this Letter. A family of sufficient conditions are given for checking global exponential stability and the existence of periodic solutions of DCNNs. These results have important leading significance in the design and applications of globally stable DCNNs and periodic oscillatory DCNNs. Our results extend and improve some earlier publications.

MSC: 34K20; 34K13; 92B20

Keywords: Cellular neural network; Exponential stability; Periodic solution; Coincidence degree

1. Introduction

It is well known that a cellular neural network (CNN) is formed by many units called cells, and that a cell contains linear and nonlinear circuit elements, which typically are linear capacitors, linear resistors, linear and nonlinear controlled sources, and independent sources. The structure of a CNN is similar to that found in cellular automata. Namely, any cell in a CNN is connected only to its neighbor cells. The circuit diagram and connection pattern modelling a CNN can be found in [1,2]. Nowadays, cellular neural networks (CNNs) are widely used in signal and image processing, associative memories, pattern classification (see, for instance, [2–6]). In the last decades,

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dynamic behaviors of CNNs have been intensively studied because of the successful hardware implementation and their widely applications. See, for example, [4–24] for stability and periodicity analysis for CNNs.

As pointed out in [4], processing of moving images requires the introduction of delay in the signals transmitted among the cells. Studying of exponential stability and existence of periodic solutions of cellular neural networks with delays (DCNNs) has been difficult. To the best of our knowledge, few authors [7–11,14,15] have considered global exponential stability and periodic solutions for the DCNNs with unbounded activation functions and time-varying delays. However, in some applications, one requires to use unbounded activation functions. For example, when neural networks are designed for solving optimization problems in the presence of constraints (linear, quadratic, or more general programming problems), unbounded activations modelled by diode-like exponential-type functions are needed to impose constraints satisfaction. The extension of the quoted results to the unbounded case is not straightforward. Different from the bounded case where the existence of an equilibrium point is always guaranteed [22], for unbounded activations, it may happen that there is no equilibrium point (see [23]). When considering the widely employed piecewise-linear neural networks (see [1,3]), infinite intervals with zero slope are present in activations, it is of great interest to drop the assumptions of strict increase and continuous first derivative for the activation. Morita [24] showed that the absolute capacity of an associative memory model can be remarkably improved by replacing the usual sigmoid activation functions with nonmonotonic activation functions. Therefore, it seems that for some purposes, nonmonotonic (and not necessarily smooth) functions might be better candidates for neuron activation in designing and implementing an artificial neural network. On the other hand, the delays in artificial neural networks are usually time-varying, and sometimes vary violently with time due to the finite switching speed of amplifiers and faults in the electrical circuit. They slow down the transmission rate and lead to some degree of instability in circuits. Therefore, fast response must be required in practical artificial neural-network designs. The technique to achieve fast response troubles many circuit designers. So, it is more important to investigate the dynamic behaviors of neural networks with time-varying delays.

Keeping this in mind, in this Letter, without assuming the boundedness, monotonicity, and differentiability of activation functions, we consider a class of DCNNs described by the following delay differential equations:

$$\dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j\big(x_j(t)\big) + \sum_{j=1}^{n} b_{ij}(t) f_j\big(x_j\big(t - \tau_{ij}(t)\big)\big) + I_i(t), \tag{1.1}$$

$i = 1, 2, \ldots, n$, where n corresponds to the number of cells in a neural network; $x_i(t)$ denotes the potential (or voltage) of cell i at time t; $f_i(\cdot)$ denotes a nonlinear output function; $I_i(t)$ denotes the ith component of an external input source introduced from outside the network to cell i at time t; $c_i(t)$ denotes the rate with which cell i resets its potential to the resting state when isolated from other cells and inputs at time t; $a_{ij}(t)$ and $b_{ij}(t)$ denotes the strengths of connectivity between cell i and j at time t respectively; $\tau_{ij}(t)$ corresponds to the time delay required in processing and transmitting a signal from the jth cell to the ith cell at time t.

Obviously, model (1.1) is one of the most popular and typical neural network models. Some other models, such as continuous BAM (bidirectional associative memory) networks and Hopfield-type neural networks, are special cases of the network model (1.1) (see, for instance, [20,21,28]).

The purpose of this Letter is to derive some new and simple sufficient conditions for the global exponential stability and the existence of periodic solutions of model (1.1). Our results extend and improve some earlier publications.

For the sake of convenience, two of the standing assumptions are formulated below:

(H_1). $|f_j(u)| \leqslant p_j|u| + q_j$ for all $u \in R$, $j = 1, 2, \ldots, n$, where p_j, q_j are nonnegative constants.

(H_2). There exist nonnegative constants p_j, $j = 1, 2, \ldots, n$, such that $|f_j(u) - f_j(v)| \leqslant p_j|u - v|$ for any $u, v \in R$.