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Numerical analysis of the effects of cross-correlated quantum noise terms in a single-mode laser

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Abstract

The effects of cross-correlation between the real and imaginary terms of quantum noise in a single-mode laser are investigated through robust numerical simulations. In comparison with the previous theoretical analyses [S.Z. Ke, et al., Phys. Lett. A 281 (2001) 113], in which the closed form of the laser intensity Langevin equation was derived and a first-order-like transition was predicted under approximative phase locking procedure, some interesting results are found as follows: (i) the laser phase locking undergoes a process as the cross-correlation strength varies from zero to unity. When the quantum noise terms are perfectly correlated, the phase can be universally locked to the most stable states. The mechanism of phase locking is revealed. (ii) The prediction of a first-order-like transition in the single-mode laser is confirmed by the numerical simulations, and the validity of the approximative phase locking procedure is evaluated.

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Keywords: Single-mode laser; Quantum noise; Cross-correlation; Phase lock

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1. Introduction

The statistical properties of a single-mode laser with complex quantum noise have been extensively studied through experimental measurements and theoretical analyses [1–5]. Most of the work take it for granted that the real and imaginary terms of the quantum noise are independent although both noise terms are originally come from the same source. However, early in 1984 Singh showed that the cross-correlation between the quantum noises for a homogeneously broadened two-mode ring laser at line center leads to nonzero contribution [3]. In 1991, Fuliński pointed out that fluctuations of the complex field can be co-

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herent, and the real and imaginary parts of the complex quantum noise generally stem from a common origin in a laser, thus may be cross-correlated [6]. It is regretful no experimental evidences reveal the existence of this cross-correlation in a single-mode laser, or there may be but need exposure theoretically. It is of interest to notice that, a critical interplay of two cross-correlated noises in a quasi-isotropic laser is experimentally demonstrated recently [7]. The output of the noise generator is divided into two parts which are obviously cross-correlated. Moreover, the cross-correlation strength between the two parts can be controlled, and the perfect cross-correlation $\lambda = \pm 1$ can be chosen experimentally via the sense of the noisy magnetic field. The measured value of the cross-correlation strength is $\lambda = 0.995$ and the experimental response is in good agreement with the theoretical predictions. The reported lever-assisted two-noise stochastic resonance can widen the potentialities of stochastic resonance [8].

The steady-state fluctuations of two-dimensional single-mode gas laser with cross-correlation between the real and imaginary parts of the quantum noise have been investigated theoretically by Zhou et al. [9]. They presented a trial solution for the probability distribution function of the laser intensity and phase. Also, the mean, normalized variance and skewness of the laser intensity as well as the phase were calculated through a potential approximation. However, the stationary intensity distribution (SID) and stationary phase distribution are not obtained, respectively. Recently, we have proposed a approximative phase locking procedure [10] by virtue of the technique developed by Sargent III et al. [1]. The closed form of the laser intensity Langevin equation under stable phase lock is derived, and the analytical expression of the SID for the laser is obtained. A first-order-like phase transition for the laser is predicted. However, most of the previous work always avoid the detailed analysis of the phase in the laser field in order to obtain analytical results which indeed need experimental or numerical verifications [10–12].

In the present Letter, we will directly investigate the effects of the cross-correlation between the real and imaginary parts of quantum noise on the phase in a single-mode laser through numerical simulations for the first time, which will show the statistical characteristics of the laser phase based on the phase evolution with different noise-cross-correlations. In Section 2, we present the stochastic trajectories and the stationary probability distributions of the phase with the change of noise-cross-correlation strength. In Section 3, the mechanism of phase locking is explored. The validity of the approximative phase locking procedure is checked with the computer experiment, and discussions are drawn in the last section.

2. The effects of noise-cross-correlation on phase

According to the third-order Lamb theory, the cubic model of a single-mode laser is one of the fundamental models of stochastic dynamics in practical laser systems, which is described by the following Langevin equation [1–5],

$$\dot{\mathbf{E}} = a_0\mathbf{E} - A|\mathbf{E}|^2\mathbf{E} + \mathbf{q}(t), \tag{1}$$

where $\mathbf{E}$ is the complex laser field, a_0 and A represent net gain and self-saturation coefficient, respectively. The random variable $\mathbf{q}(t)$ represents the intrinsic complex quantum noise due to spontaneous emission which is assumed to be the Gaussian white noise term with zero mean and correlation,

$$\langle q_i(t)q_j(s)\rangle = \epsilon_{ij} Q\delta(t-s), \tag{2}$$

$$\epsilon_{ij} = \begin{pmatrix} 1 & \lambda \\ \lambda & 1 \end{pmatrix}, \tag{3}$$

where $i, j = 1, 2$ represent the real and imaginary parts of quantum noise which arise from a common origin, thus may be cross-correlated, the correlation coefficient λ measures the degree of correlation between $q_1(t)$ and $q_2(t)$, and $|\lambda| \leqslant 1$. $\lambda \to 0$ means that the quantum noise is a circular complex Gaussian noise, and the real and imaginary parts of the noise are independent of each other; whereas $|\lambda| \to 1$ implies that the real and imaginary parts of the noise are (positively or negatively) perfectly correlated [13]. A two-dimensional model stochastically equivalent to the cubic laser model reads in polar coordinate variables ($\mathbf{E} = re^{\mathbf{i}\phi}$),

$$\frac{dr}{dt} = a_0 r - Ar^3 + \epsilon_r(t), \tag{4}$$

$$\frac{d\phi}{dt} = \frac{1}{r}\epsilon_\phi(t), \tag{5}$$